NINE

GEOMETRICALL

EXERCISES,

FOR

Young Sea-men,

And others that are ſtudious in

MATHEMATICALL PRACTICES:

Containing IX *particular* TREATISES, *whoſe* Contents *follow in the next Pages.*

All which EXERCISES are *Geometrically* performed, by a Line of *Chords* and *equal Parts*, by waies not uſually known or practiſed. Unto which the *Analogies* or *Proportions* are added, whereby they may be applied to the *Chiliads* of *Logarithms*, and *Canons* of Artificiall *Sines* and *Tangents*.

By *William Leybourn*, Philomath.

LONDON,

Printed by *James Fleſher*, for *George Sawbridge*, living upon *Clerken-well-green*. *Anno Dom.* 1669.

457.464

NINE GEOMETRICALL EXERCISES, FOR Young Sea-men,

And others that are studious in MATHEMATICALL PRACTICES:

Containing IX particular TREATISES, whose Contents follow in the next Page.

All which EXERCISES are Geometrically performed, by a Line of Chords and equal Parts, by wares not exactly known or practised. Unto which the Analogies or Proportions are added, whereby they may be applied to the Chords of Logarithms, and Canons of Artificial Sines and Tangents.

By William Leybourn, Philomath.

LONDON,

Printed by James Flesher, for George Sambridge, living upon Clerken-well-green. Anno Dom. 1669.

THE CONTENTS

Of the ſeverall

EXERCISES.

I.

THE First Exerciſe *contains ſuch* Geometricall Propoſitions *and* Theorems *as are neceſſary to be known and practiſed for the more eaſie underſtanding of the ſubſequent* Exerciſes.

II.

The Second Exerciſe *contains the* Doctrine *of the* Dimenſion *of* Right-lined Triangles, *both* Right *and* Oblique-angled, *by* Protraction, *by a* Line *of* Chords *and* equal Parts.

III.

The Third Exerciſe *contains the* Doctrine *of the* Dimenſion *of* Sphericall Triangles, *both* Right *and* Oblique-angled, *by a* Line *of* Chords *onely.*

IV.

The Fourth Exerciſe *is a plain and eaſie* Method *of* Projecting the Sphere in Plano; *whereby the* Sides *and* Angles *of* Sphericall Triangles *are naturally laid down as they are in the* Sphere *it ſelf. By which the* Nature *of them is diſcovered, and their* Sides *and* Angles *meaſured with* ſpeed *and* exactneſs *upon the* Projection *it ſelf.*

V.

The Fifth Exerciſe *declares the* Variety *of* Sphericall Pro-

blems *which do naturally arise out of every* Spherical Triangle, *both* Right *and* Oblique-angled, *and which are resolvable thereby. Described as they are perspicuous in the* Projection.

VI.

The Sixth Exercise *contains such* Astronomicall Propositions *as are of frequent Use in the Practice of* Navigation. *All which* Propositions *are resolved by measuring the* Sides *and* Angles *of* Spherical Triangles *upon the* Projection.

VII.

The Seventh Exercise *shews how the foregoing* Astronomicall Propositions *may be applied to Practice in the Art of* Navigation: *As to find the* Latitude, Hour, Variation *of the* Compass, *&c.*

VIII.

The Eighth Exercise *contains such* Geographicall Propositions *as concern the finding of the* Distance *of* Places *upon the* Terrestrial Globe *in any Position, both by* Trigonometricall Calculation *and* Geometricall Projection.

IX.

The Ninth Exercise *shews the Use of* Right-lined Triangles *in the Practice of* Navigation; *whereby severall* Nauticall Questions *are resolved, many* Problems *in* Sailing, *both by the* Plain *and* Mercator's Chart, *performed by* Protraction, Calculation, *and wrought upon the* Chart *it self.*

These are the General Heads *of the Nine* Exercises: *All which Particulars you will find in the following* Treatises, *with addition of what is here recited: And the whole Work of all the* Exercises *performed by waies not usually known or practised.*

To

To the Reader.

HAving heretofore publiſhed to the World ſeverall *Treatiſes* tending to the Practice of the *Sciences Mathematicall*, as, my *Arithmetick* in Four Parts, 1. *Vulgar*, 2. *Decimal*, 3. *Inſtrumental*, and 4. *Algebraicall*; my *Arithmeticall Recreations*; my *Uſe of the Line of Proportion made eaſie*; *the Uſe of Napiers Bones*; my *Compleat Surveyor*; my *Platform for Purchaſers*, *Guide for Builders*, and *Mate for Meaſures*, &c. All which have found good acceptance in the World, as may appear by the ſeverall *Editions* which ſome of them have paſſed the Preſs: I thought good at length to publiſh ſomething that might be *uſefull* and *beneficial* for my *Country-men*, *Engliſh Navigators*; hoping that I ſhall receive from them (for the pains I have already, and hereafter ſhall take for their good and advantage) no leſs Encouragement by their acceptance of what (out of free good will) I tender to them, then I have already from others, for what I have made publick for their Uſe and Service.

In the *Nine* following *Treatiſes* I have furniſhed the *Young Sea-man* with ſuch practical *Exerciſes* as tend wholly to his *Employment* at *Sea*: beginning, firſt, with ſome few *Geometricall Rudiments* that are abſolutely neceſſary for him to underſtand and be perfect in the Practice of, before he launch farther into the *Mathematicall Ocean*.

From theſe *Elements*, I proceed to the *Geometricall Dimenſion* of *Triangles* both *Plain* and *Sphericall*, upon the *Solution* where-

whereof all the *Practick Parts* of the *Mathematicks* do depend. For on the *Solution* of *Right-lined Triangles*, all *Propositions* which concern *Longimetria*, or the Measuring of *Heights*, *Depths* and *Distances*, either upon the *Land* or at *Sea*; all in *Planometria*, as Measuring of *Land*, *Board*, *Glass*, *Pavements*, *Hangings*, &c. all in *Striometria*, or the Mensuration of *Solids*, such as *Stone*, *Timber*, and the like; all Problems of *Sailing*, both by the *Plain Sea-Chart*, and also by that of *Mercator's* Projection; all that concerns *Fortification*, *Gunnery*, &c. have their dependence.—And by the *Solution* of *Spherical Triangles*, *Problems* in *Astronomie*, *Geography*, *Sailing by the Arch of a great Circle*, *Dialling*, Measuring of the Distance of *Stars*, *Comets*, &c. are to be resolved.

Trigonometria, therefore, being the *Basis* or *Foundation* upon which the forementioned *Superstructures* are *erected*; I chose first to begin with that. And how I have performed it, and to what Uses I have applied it, which tend to the Practice of *the Art of Navigation*, the *Contents* of the severall *Exercises* will (in part) declare.

In the *Geometricall resolving* of *Triangles* both *plain* and *spherical*, I have gone in a *Path* not before *beaten*. And therefore, if I have at any time gone out of my way, either by *deficiency* or *redundancy*, I hope the *Reader* will pardon me. I am sure the ingenious *Mathematician* (who (at first sight) will discover the manner and waies of Working, and also the Ground and Demonstration thereof) will not *carp* or *cavil* at, but rather inform *me* or the *Reader* of any such *Deviation*. But in the Pursuance of every *Exercise*, I have therein retained such a *Method*, that I could not well step aside. And for the certainty of the manner of the *Geometricall Operations*, (if they be carefully performed) I dare aver that you may (if the Radius of your Line of Chords be but 4 or 5 *Inches*) come within a few Minutes of Calculation; which will be sufficient for the *Sea-man's Use*, for him thereby to resolve such *Problems* in *Astronomie*, *Geography* or *Navigation*, as at any time he shall (in his Practice)

have

have occaſion for. And that Proof or Triall may be made thereof, (though I could in a few Lines demonſtrate the nature and manner of Working) I have to every *Caſe*, *Propoſition* and *Problem* in the whole *Work* contained, added the *Analogies* or *Proportions* by which they may be *ſolved* by the *Canons* of *Artificial Sines* and *Tangents*, and the *Chiliads* of *Logarithms*: So that ſuch as have thoſe Tables (and know their Uſe) may work any *Propoſition* by them, and by that diſcover the Difference between *Calculation* and this *Geometricall* way of *Operation*.

And that the *young Sea-man* may not (at any time) be to ſeek of *Tools* to work with, (in caſe of any Miſcarriage at Sea) I have not onely ſhewed him how to *uſe*, but alſo the manner how to *make*, all ſuch *Inſtruments* as will be in uſe to him in the Practice of theſe *Exerciſes*: As of his *Line of Chords*, which is the chief; and alſo of a *Sea-Chart* according to *Mercator's* Projection, without the help of a *Table* of *Meridionall Parts*, or a *Meridian Line*, but *eaſily* and *ſpeedily* by a *Line of Chords* onely; and that either *General*, from the *Æquinoctial* towards either *Pole*; or *Particular*, for any deſigned *Navigation*.

Moreover, in the Working of the ſeverall *Queſtions* by *Mercator's Chart*, I have, at the end of every *Problem*, given an account of the Difference ariſing between Working by that and the *Plain Sea-Chart*; whereby the *Errours* of the one are clearly detected, and the *Verity* of the other diſcovered.

I might have proceeded farther, to the performance of other things by the *Line of Chords* onely; as to ſhew the manner of *Sailing* by the *Arch of a great Circle*: but that, with ſome other things tending to *Navigation*, I ſhall ſhortly make publick in a *Treatiſe* formerly written by my ſelf and Mr. *Vincent Wing*, now in my hands, and almoſt ready for the *Preſs*, and wholly deſigned for the Uſe of *Sea-men*.—And for farther Uſe of this Line, I have in a Treatiſe (now in the Preſs) taught the manner how to delineate all manner of *Sun-Dials* by *projecting* of the *Sphere in Plano*, whereby not onely the *Making*, but the *Reaſon* alſo, of

Dials is discovered. To which Treatise (if my present occasions will permit) I shall adde both an Arithmeticall and an Instrumentall way of *Dialling* also.

Thus, friendly Reader, at present, I present thee with This; and in a few Months thou shalt (God willing) participate of those above mentioned. All which I hope will be as kindly accepted, as they are freely tendered by

Will. Leybourn.

Arts

Arts and Sciences MATHEMATICALL
Profeſſed and Taught by the Authour.

Arithmetick, { In Whole Numbers, and Fractions. / In Decimals, and by Logarithms. / Inſtrumentally, by Decimal Scales, Napiers Bones: and to extract the Square and Cube Roots by Inſpection.

Geometrie: { The Principles thereof with the { Practice, and Demonſtration.

Aſtronomie: { The Deſcription of the Circles of the Sphere. / The Uſe of the Globes, { Celeſtiall, and Terreſtriall. / To project the Sphere in *Plano* upon any Circle, { Right, or Oblique.

And upon theſe Foundations the following Superſtructures.

The Uſe of Geometricall Inſtruments, in the Practice of

- *Longimetria*, or the Menſuration of { Heights, Depths, Diſtances, } as of { Trees, Towers, &c. / Mines, Wells, Deſcents, &c. / Churches, Towers, &c.
- *Planometria*, or the Menſuration of { Board, Glaſs, Pavement, Tiling, &c. } Or any other Superficies.
- *Stereometria*, or the Menſuration of { Timber, growing or squared. / Stone, regular or irregular. / Cask, commonly called Gageing.
- *Geodeſia*, or the Meaſuring of Land divers waies, and by ſeverall Inſtruments; to draw the Plot of a whole Mannor or Lordſhip; to caſt up the Content thereof; and to beautifie the ſame with all neceſſary Ornaments thereunto belonging.

Trigonometria: { Or, the Menſuration of Triangles, both { Plain, and Spherical. / The Application thereof, in the ſolution of Problems in { Geometry. / Aſtronomie. / Geographie. / Navigation. / Fortification. / Dialling, &c.

Navigation: { The Principles thereof, and the manner of Sailing by { The Plain Sea Chart. / *Mercator's* Chart. / The Arch of a great Circle.

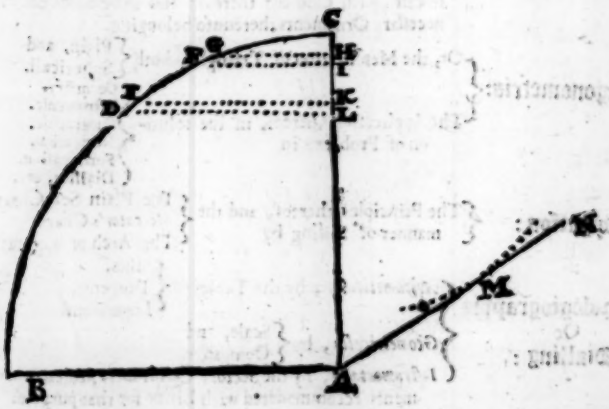
Gnomonologiographia Or Dialling: { *Arithmetically*, by the Tables of { Sines. / Tangents. / Logarithms. / *Geometrically*, by { Scale, and Compaſſes. / *Inſtrumentally*, by the Sector, Quadrants, Scales, and other Inſtruments accommodated with Lines for that purpoſe.

You may hear of him at Mr. *Hayes's* at the *Croſs-daggers* in *Moor-fields*.

ADVERTISEMENT.

AND here I thought good to give notice, That if any Gentlemen ſtudious in the Mathematicks have or ſhall have occaſion for Inſtruments thereunto belonging, or Books to ſhew the Uſe of them, they may be furniſhed with all ſorts uſefull both for Sea or Land, either in Silver, Braſs or Wood, by Walter Hayes, *at the* Croſs-daggers *in* Moor-fields, *next door to the* Pope's-head *Tavern; where they may have all ſorts of Maps, Globes, Sea-plats, Carpenters Rules, Poſt and Pocket-Dials for any Latitude.*

This Scheme having relation to the Fourth and Fifth Caſes of Sphericall Triangles in Page 53 and 54, being caſually omitted, is here inſerted.



GEO-

GEOMETRICALL PROPOSITIONS and THEOREMS,

Neceſſary to be known and practiſed for the more eaſie underſtanding of the ſubſequent EXERCISES.

The Firſt EXERCISE.

THE ARGUMENT.

THE *following* Propoſitions, Theorems *and* Problems, *are ſuch as will come in continuall uſe in the Practice of the ſubſequent* EXERCISES, *and therefore I have here promiſcuouſly inſerted ſuch as do relate to the following* Treatiſes, *and ought throughly to be* underſtood *and often* practiſed; *by which means nothing in the enſuing* Caſes, Propoſitions *and* Problems, *will be difficult to be underſtood, but that you may gradually proceed, in your* Practice, *from* Exerciſe *to* Exerciſe, *without being referred to any other* Book *or* Books.

GEOMETRICALL PROPOSITIONS.

PROP. I.

A right Line *being given, to divide the ſame into two* equal *parts at* right Angles.

LET the right Line, to be divided, be A B.—Firſt, open your Compaſſes to any diſtance greater then the length of half the given Line; then ſetting one foot in A, with the other foot deſcribe the Arches C C, both above and below the Line A B.—Secondly, (the Compaſſes ſtill reſting at the ſame diſtance,) ſet one foot in B, and with the other deſcribe the Arches D D, cutting the former Arches C C in the Point E above, and F below the given Line A B.—Laſtly, draw the Line E F, which will divide the given Line A B in two equal parts, in the Point G, and at right Angles.

PROP. II.

Upon a right Line *given, to erect a* Perpendicular *upon any part thereof.*

LET the given Line be H K, and from the Point L let it be required to erect a Perpendicular.—Firſt, open your Compaſſes to any convenient diſtance, and ſetting one foot in the given Point L, with the other make a Mark or

Point

Point at pleaſure, as M. Then keeping the Compaſs-point in M, with the other deſcribe the Arch N N above the Point L, and alſo another Arch at O, cutting the given Line in O.——Laſtly, lay a Ruler from O to M, which will cut the Arch N N before drawn in the Point P. So a Line drawn from P to L ſhall be a Perpendicular to the given Line H K, and from the Point L.

Prop. III.

From a Point *above, to let fall a* Perpendicular *upon a* right Line *given.*

LET the Point given be Q, and the Line upon which the Perpendicular is to fall be R S.———Firſt, opening the Compaſſes to any diſtance greater then Q X, ſet one foot in Q, and with the other croſs the given Line R S in the Points T and V.—Secondly, (the Compaſſes unaltered,) ſet one foot in T, and with the other deſcribe the Arch *w w*, below the given Line R S.—— Thirdly, remove the Compaſſes to V, (being ſtill at the ſame diſtance) and croſs the Arch *w w* in the Point Z—. Laſtly, a Ruler laid from Q to Z will cut the Line R S in the Point X. So a Line drawn from Q to X ſhall be perpendicular to the given Line R S.

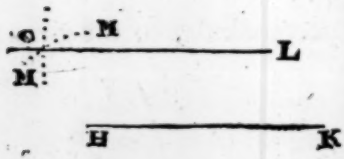
Prop. IV.

A right Line *being given, to draw another* right Line *which ſhall be* parallel *thereto at any diſtance required.*

LET AB be a Line given, and let it be required to draw another right Line which ſhall be parallel thereunto, and at the diſtance of the length of the Line C.——Firſt, take the length of the Line C in your Compaſſes, and ſetting one foot towards one end of the given Line, as at D, deſcribe the Arch E.——Secondly, ſet one foot of the Compaſſes towards the other end of the given Line, as at F, and deſcribe the Arch G.——Laſtly, lay a Ruler to the Arches E and G, ſo that the Ruler onely touch the Arches, and not cut or croſs them in any part. So a Line drawn thereby ſhall be parallel to the given Line A B, and at the diſtance of the Line C.

Prop. V.

A right Line *being given, to draw another* right Line parallel *thereunto, which ſhall paſs through a given* Point.



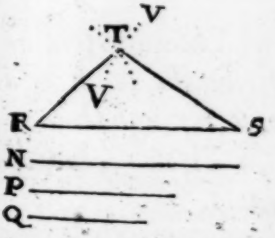
LET the given Line be H K, to which let it be required to draw a parallel Line, which ſhall paſs through the Point L.—Firſt, take in your Compaſſes the diſtance K L, and ſetting one foot of the Compaſſes in H, with

H, with the other deſcribe the Arch M M.———Secondly, take the Line H K in your Compaſſes, and ſetting one foot in the given Point L, with the other croſs the Arch M M in the Point O. So a Line drawn from L to O ſhall be parallel to the given Line H K, and ſhall paſs through the Point L.

Prop. VI.

Three right Lines *being given, to make a* Triangle, *whoſe* three Sides *ſhall be equal to the* three *given* Lines.

LET the three Lines given be N, P, Q.———First, take the Line N in your Compaſſes, and lay that down from R to S.———Secondly, take the Line P in your Compaſſes, and ſetting one foot in S, with the other deſcribe the Arch V V.———Thirdly, take the Line Q in your Compaſſes, and ſetting one foot in R, with the other croſs the Arch V V, in the Point T.———Laſtly, draw the Lines T R, and T S. So ſhall you have conſtituted the Triangle T R S, whoſe three Sides are equal to the three given Lines, N, P, Q.

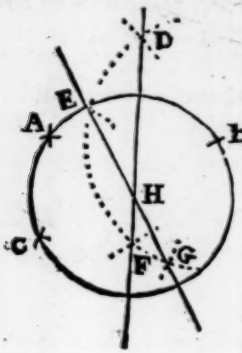


Prop. VII.

Three Points (*which lie not in a ſtraight Line*) *being given, to finde the* Centre *of a* Circle, *which being deſcribed ſhall paſs through the three given* Points.

LET the three given Points be A, B, C.———First, open your Compaſſes to any diſtance greater then half the diſtance between A and B: and ſetting one foot in B,

with the other deſcribe the Arch G D. Then remove the Compaſſes, ſet one foot in A, and with the other croſs the former Arch in the Points D and F, and draw the Line D F.——Secondly, (the Compaſſes ſtill continuing at the ſame diſtance,) ſet one foot in the Point C, and with the other croſs the Arch (before drawn) in the Points E and G, and draw the Line E G, croſſing the other Line D F in the Point H. So ſhall H be the Centre of the Circle, which being deſcribed ſhall paſs directly through the three given Points, A, B, C.

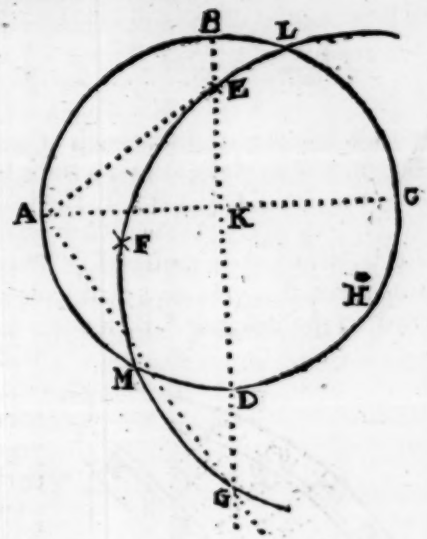


Prop. VIII.

Two Points *within any* Circle *being given, how to deſcribe the* Arch *of another* great Circle *which ſhall paſs through thoſe two given* Points, *and alſo divide the Circumference of the given* Circle *into two equal parts.*

LET the two Points given be E and F, within the Circle A B C D.——Firſt, through either of them (as through E) draw the right Line E D, paſſing through the Centre of the Circle at K.——Secondly, draw the Line A C at right Angles to B D; ſo ſhall the Circle be divided into four equal parts or Quadrants, by the Lines A C and B D. ——Thirdly, draw the Line E A, and upon the Point A (by the II. Prop.) erect the Perpendicular A G, cutting the Line B D (it being extended) in the Point G; ſo have you three Points, E, F, and G, through which (by the laſt Prop.) ou may draw a Circle to paſs, whoſe Centre will be at H: upon

upon which Point if you deſcribe an Arch of a Circle, at the diſtance HE or HF, it will paſs through the two given Points E and F, and divide the Circle ABCD into two equal parts, in the Points L and M, which was required. And that this Arch, thus drawn, doth divide the Circle into two equal parts, is evident, for a Line drawn from L to M will paſs directly through the Centre K.



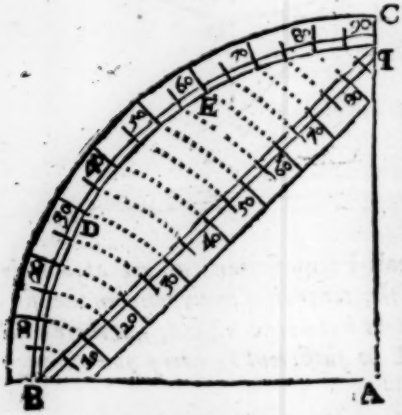
Theſe are ſuch Geometricall Propoſitions *as are abſolutely neceſſary for the working of the ſeverall Concluſions in the following* Exerciſes. *More might have been added, but theſe well underſtood and practiſed will be ſufficient to carry you through this Work.*

And now I will deſcribe unto you the making of the ſlight Inſtrument by which all contained in this Book is performed, namely, The Line of Chords, *and ſhew you the general Uſe thereof, in the* protracting *or* laying down *of* Angles *of any quantity: Or if any* Angle *be already* laid down, *to finde (thereby) the quantity thereof.*——*Then will I give you ſome few uſefull and neceſſary* Theorems, *chiefly appertaining to* Trigonometry, *or the* Solution of Triangles, *and ſo conclude this firſt* EXERCISE.

How

How to make a Line of Chords.

ACcording to the largeneſs of your *Line of Chords* you intend to make, draw a right Line, as A B, and upon the Point A (by the II. Prop.) erect the Perpendicular A C, and upon A (as a Centre) deſcribe the *Quadrant* B D E C, which you muſt divide into 90 equal parts or *Degrees*. Which that you may readily doe, your Compaſſes being opened to the diſtance A B, ſet one foot in B, and the other will reach to E; alſo ſet one foot in C, and the other will reach to D: ſo is your *Quadrant* divided into three equal parts, each part containing 30 degr. This done, divide each of theſe three parts into three more; ſo ſhall you have divided your *Quadrant* into 9 equal parts, each containing 10 degr. and each of theſe 9 parts, being divided into halves, will contain 5 degr. and (if you make your Line large enough) you muſt divide thoſe into 5 equal parts, which you may very well doe, if the Line A B be but two inches long, as all the *Schemes* and *Figures* in theſe *Exerciſes* are drawn by a *Line of Chords* of that length.



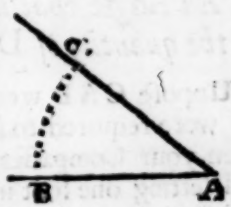
Your *Quadrant* being thus divided into 90 degr. draw the Line B C, and parallel thereto two other Lines, one pretty cloſe to B C, to contain the ſmall Diviſions, and the other at a larger diſtance, to ſet the Figures in. Now it is the Line B C which is called the *Line of Chords*, (poſſibly for this Reaſon, the

the Arch or Ark B D E C repreſenting *Arcus*, a *Bow*, and B C the *String* or *Chord* thereof) the diviſions whereof are to be transferred from the degrees of the *Quadrant* B D E C, in this manner.—— Firſt, ſetting one foot of your Compaſſes in B, extend the other to 80 degr. in the *Quadrant*, and from the diviſion of 80 degr. in the *Quadrant* draw the Arch 80, 80, which will cut the *Chord-Line* in 80; doe ſo with 70, 60, 50, *&c.* and the like with every fifth degree, as you ſee in the Figure. And if your Line be very large, you may doe ſo to every ſingle degree, and part of a degree. And by this means have you reduced the degrees of the *Quadrant* B D E C to the ſtraight Line B C, more commodious to be ſet upon a Ruler, then the crooked Arch B D E C.

The Uſes *of the* Line of Chords.

THE *Uſes* of this Line are principally two. The *one* is, To *protract* or *lay down* upon Paper an *Angle* of any quantity (that is, of any number of *degrees*) required.—— The *other Uſe* is, If an *Angle* be already *protracted* or *laid down*, to finde how many *degrees* and parts of a *degree* it containeth. ——In both which I would have the Reader very perfect, becauſe very much contained in this Book hath dependence thereupon.

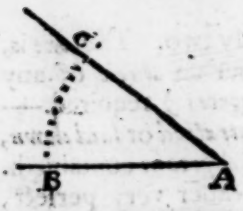
And here it will be neceſſary that I give you the Definition of an Angle. Know therefore that an Angle is the Inclination or bowing of two right Lines the one to the other.——As the two right Lines C A and B A incline the one to the other, and touch or meet each other in the Point A, in which Point, by reaſon of the inclination of the ſaid Lines, is made the Angle C A B.



And here note that an Angle is commonly ſigned by three Letters, the middlemoſt whereof ſignifies the angular Point. As in this Figure, when we ſay the Angle C A B, you are to underſtand the very Point at A.

I. *How to* protract *(or lay down upon Paper) an* Angle *containing any number of* Degrees *and* Minutes *by the* Line of Chords.

DRaw a right Line at pleaſure, as A B, and from the Point A let it be required to protract or lay down an Angle containing 40 degrees.——Firſt, open your Compaſſes alwaies to 60 degr. of your *Line of Chords*, (which is equal to the Line A C of the *Quadrant*,) and with this diſtance, ſetting one foot of the Compaſſes upon the Point A, with the other foot deſcribe the Arch B C.——Secondly, take in your Compaſſes 40 degr. (which is the quantity of the Angle to be laid down) out of the *Line of Chords*, from the beginning thereof, and ſetting one foot in B, the other will reach to C upon the Arch: wherefore through the Point C draw the Line C A. So ſhall the Angle at A contain 40 degr. as was required.



II. *An* Angle *that is already* protracted, *how to finde the quantity of* Degrees *it containeth.*

SUppoſe C A B were an Angle already *protracted*, and it were required to finde the quantity thereof.——Firſt, open your Compaſſes to 60 degr. of your *Line of Chords*, and ſetting one foot in A, (the angular Point) with the other deſcribe the Arch B C.——Secondly, take in your Compaſſes

paſſes the diſtance between B and C, which diſtance apply to your *Line of Chords*, (by ſetting one foot in the beginning thereof) and you ſhall finde the other to fall upon 40 degr. which is the quantity of the Angle at A.

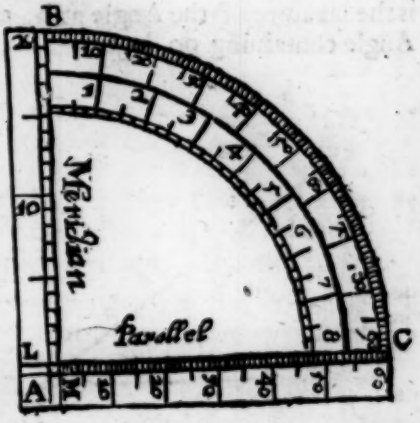
Thus have you the *Uſes* of your *Line of Chords* in *protracting* and finding the quantities of *Angles*. And now it will not be impertinent, if in this place I ſhew you how *Angles* may be *protracted* and laid down, and alſo their quantities found, by an *Inſtrument* which I ſhall make uſe of towards the end of this *Book*, which I call a *Protracting Quadrant*.

Its Deſcription.

IT is no other then a *Quadrant* made upon a piece of very thin Braſs, and divided into 90 degr. the Braſs being cut away cloſe to the diviſions of the degrees on the out-ſide, and alſo the hollow within, ſo that there remains nothing but the *Limb* and the two *Sides*, as you may diſcern by the Figure. In the *protracting* or laying down of *Angles*, and in finding of the quantity of *Angles* already laid down, this is

Its Uſe.

SUppoſe you were to finde the quantity of the Angle C A B. Hold a *Pin* or *Needle* upon the angular Point at A, to which bring the Centre of your *Quadrant*, (noted alſo with A) and there turn it about, till the *Meridian Line* thereof, A B, lie upon the Line A B of the *Angle*: then ſee under what degrees of the *Quadrant* the Line A C



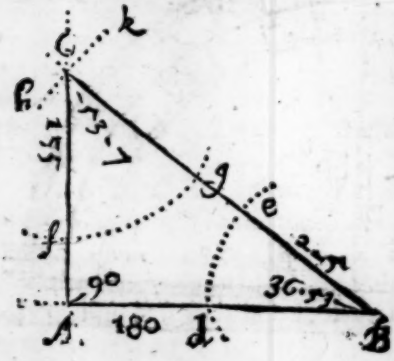
lieth, which you ſhall find to lie juſt under 40 degr. And ſuch is the quantity of the Angle C A B.—And if upon the Line B A you were to protract ſuch an Angle of 40 degr. Lay the *Meridian* Line of the *Quadrant* upon the Line A B, (the Centre of the *Quadrant* upon the Point A) and with your Needle make a prick or point juſt againſt 40 degr. of the *Quadrant*'s limb. So a Line drawn from A through this Point ſhall make an Angle of 40 degr.

Trigonometricall Theorems.

1. A Triangle is a Figure conſiſting of three Sides, and as many Angles; as is the Figure C A B.

2. Any two Sides of a Triangle are called the Sides of the Angle contained by them; as the Sides C B, and A B, are the Sides containing the Angle C B A.

3. The meaſure of an Angle is the quantity of the Arch of a Circle, deſcribed upon the angular Point, and cutting both the Sides containing the Angle. As in the Triangle A C B, the Arch *f g* is the meaſure of the Angle at C, the Arch *d e* is the meaſure of the Angle at B, and the Angle at A is a right Angle containing 90 degr.



4. A Degree is the 360. part of any Circle. Therefore,

5. A Semicircle contains 180 degr. And

6. A Quadrant (or right Angle) contains 90 degr.

7. The Complement of an Angle leſs then 90 degr. is ſo much as that Angle wanteth of 90 deg.

As

As the Angle A C B of the Triangle containeth 53 degr. 7. min. the Complement whereof is 36 degr. 53 min. which is ſo much as 53 degr. 7 min. wanteth of 90 degr. For if you ſubtract 53 degr. 37 min. from 90 degr. (or from 89 degr. 60 min. for eaſe in ſubtracting) the remainder will be 36 degr. 53 min.

8. The Complement of an Angle to a Semicircle is ſo much as that Angle wanteth of 180 degr. So the Angle C being 53 degr. 7 min. take 53 degr. 7 min. from 180 degr. (or from 179 degr. 60 min.) and the remainder will be 126 degr. 53 min. which is the Complement of the Angle C to 180 degr.

9. An Angle is either right, acute, or obtuſe.

10. A Right Angle is that whoſe meaſure is 90 degr. or a Quadrant.

11. An Acute Angle is leſs then a right Angle, and alwaies contains leſs then 90 degr.

12. An Obtuſe Angle is greater then a right Angle, and alwaies contains more then 90 degr.

13. A Triangle is either right-angled, or oblique-angled.

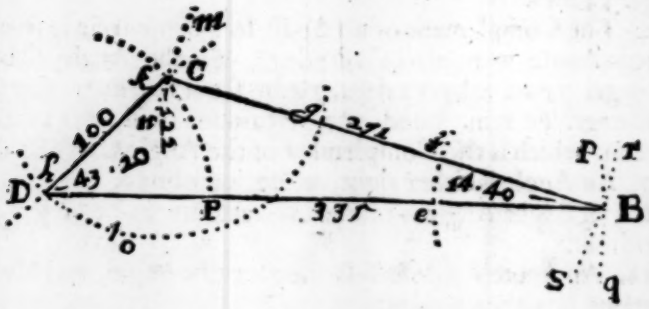
14. A right-angled Triangle is ſuch a Triangle as hath one right Angle. As the Triangle C A B hath one right Angle, namely, that at A, which containeth juſt 90 degr.

15. In every right-angled Triangle, that Side which ſubtendeth (or lieth oppoſite to) the right Angle is called the *Hypotenuſe*; and of the other two Sides the one is called the Perpendicular, and the other the Baſe, at pleaſure: But moſt commonly the ſhorter ſide is called the Perpendicular, and the longer the Baſe. Thus in the Triangle C B A, B C is the Hypotenuſe, C A the Perpendicular, and A B the Baſe.

16. In every right-angled Triangle, if you have one of the acute Angles given, the other is alſo given, it being the Complement thereof to 90 degr. As in the Triangle C A B, if you have the Angle at C 53 degr. 7 min. given, you have alſo the Angle at B given, it being the Complement of that at C to 90 degr. wherefore take 53 degr. 7 min. from

90 degr. and there will remain 36 degr. 53 min. which is the quantity of the Angle at B.

17. In all right-lined Triangles whatſoever, (either right-angled or oblique-angled) the three Angles together are



equal to two right Angles, or contain 180 degr. Therefore, if you have any two Angles of a Triangle given, you have alſo the third given, it being the Complement of the other two to 180 degr.——Thus in the Triangle C D B, if there were given the Angle C D B 43 degr. 20 min. and the Angle C B D 14 degr. 40 min. I ſay, by conſequence you have the third Angle D C B alſo given, it being the Complement of the other two to 180 degr. For the two given Angles B D C 43 degr. 20 min. and C B D 14 degr. 40 min. being added together, make 58 degr. which being taken from 180 degr. there will remain 122 degr. the quantity of the obtuſe Angle D C B.

18. In all Triangles whatſoever, the Sides are in proportion one to the other as the Sines of the Angles oppoſite to thoſe Sides. So in the Triangle C D B, the Sine of the Angle at D is the proportion to the Side C B, which is oppoſite to it; as the Sine of the Angle at B is to the Side C D, or the Angle at C to the Side D B.

THE

THE *SOLUTION* Of Right-lined TRIANGLES By the *LINES* of *EQUAL PARTS, & CHORDS.*

The Second EXERCISE.

WHAT a Line of equal parts, *and a* Line of Chords, *is, is not unknown to any conversant in the leaſt with* Mathematicall *Practices. For a* Line of equal parts *is onely a Line of any length equally divided into any number of equal parts, as* 10, 20, 30, 60, 100, 1000, &c.——*And a* Line of Chords *is no other then the Degrees of a Quadrant, or quarter of a Circle, (which contains* 90 *degr.) transferred from thoſe degrees to a ſtraight Line; as in the foregoing Exerciſe I have ſhewed. Theſe Lines*

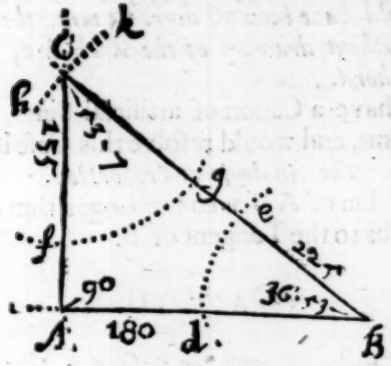
Lines of equal parts *and* Chords, *being put upon a plain* Ruler, *(which any that make* Mathematicall Inſtruments *know how to doe) will, by the* Precepts *following, meaſure all manner of* Triangles, *whether* plain *or* ſpherical. *But before I come to the Solution of the ſeverall* Caſes *appertaining to* Triangles, *I would have you take notice, That every* Triangle, *whether* plain *or* ſpherical, *conſiſteth of ſix parts, namely, of three* Sides, *and as many* Angles; *any three of which being given, a fourth may be found. Now in the reſolving of* right-lined Triangles, *if they be* right-angled, *I call the Side oppoſite to the* right Angle *the* Hypotenuſe; *and of the other Sides, comprehending the* right Angle, *I call the longer of them the* Baſe, *and the ſhorter, the* Perpendicular.——*But in the Solution of* oblique-angled plain Triangles, *I call the longeſt Side of the* Triangle *the* Baſe; *and the other two, the two* Sides, *without any other diſtinction of Denomination or Name. Theſe things being premiſed, I come now to the Solution of* plain Triangles, *both* right *and* oblique-angled. *And*

I. *Of Right-angled plain Triangles.*

THE Triangle which I ſhall make uſe of in the ſeverall Caſes belonging to a *right-angled plain Triangle* ſhall be this following, C A B, in which

		parts.
A B, the Baſe,	contains	180
C A, the Perpendicular,		135
C B, the Hypotenuſe,		225
And		deg. m.
A, the right Angle,	contains	90——00
C, the Angle at the Per.		53——07
B, the Angle at the Baſe,		36——53

CASE



Case I.

The Baſe B A 180, *and the Perpendicular C A* 135, *being given, to finde the Angles B and C.*

Draw a Line A B, and from your Scale of equal parts take 180, and ſet them from A to B; then on the Point A raiſe the Perpendicular A C, and becauſe it contains 135, take 135 parts from your Scale of equal parts, and ſet them from A to C; then draw the Line C B: which three Lines will conſtitute the Triangle C A B.

Now to finde the Angles C and B, take in your Compaſſes 60 degr. of your Line of Chords, and ſetting one foot in C, deſcribe the Arch *f g*; alſo ſetting one foot in B, deſcribe the Arch *d e*: then take in your Compaſſes the diſtance from *f* to *g*, which meaſured upon your Line of Chords will reach from the beginning thereof to 53 degr. 7 min. and ſuch is the quantity of the Angle at C.——In like manner take the diſtance between *d* and *e* in your Compaſſes, that diſtance applied to your Line of Chords will reach to 36 degr. 53 min.

——Or, *when you had found the quantity of the Angle C to be*

53 *degr.* 7 *min. if you had ſubtracted that from* 180 *degr. the remainder would have been* 36 *degr.* 53 *min. the quantity of the Angle at* B, *without drawing of the Arch* d e, *and meaſuring it upon your Chord.*

For ſuch as have a Canon of artificial Sines, Tangents and Logarithms, and would reſolve this Caſe by them, this is

The Analogie or Proportion.

As the Logarithm of A B is to the Logarithm of A C,
So is the Radius to the Tangent of B.

CASE II.

The Hypotenuſe C B 225, *and the Baſe A B* 180, *being given, to finde the Angles B and C.*

DRaw a right Line A B containing 180 parts of your Scale of equal parts, alſo out of the ſame Scale of equal parts take 225, your Hypotenuſe, and ſetting one foot of your Compaſſes in B, with the other deſcribe the obſcure Arch *h k*; then on the Point A raiſe the Perpendicular A C, which will cut the obſcure Arch *h k* in C; then draw the Line C B, ſo have you the Triangle C A B: then may you meaſure the quantity of the Angles at C and B as in the laſt *Caſe*. And ſo will C be 53 degr. 7 min. and B 36 degr. 53 min.

The Analogie or Proportion is,

As the Log. of C B is to the Radius,
So is the Log. of the Side A B to the Sine of C.

CASE III.

The Baſe A B 180, *the Angle C* 53 *degr.* 7 *min. and the Angle B* 36 *deg.* 53 *min. being given, to finde the Perpendicular C A.*

DRaw a right Line A B containing 180 parts of your Scale, for the Baſe of your Triangle; then taking 60 degr. from your Line of Chords, on the Point B deſcribe the Arch

d e,

d e, and (becauſe the Angle at B contains 36 degr. 53 min.) take 36 degr. 53 min. from your Chord, and ſet it from *d* to *e*, and from B, through the Point *e*, draw the Line B C. Alſo upon the Point A erect the Perpendicular A C, croſſing the Line B C in C. So have you formed the Triangle C A B. Laſtly, take the length of the Line A C in your Compaſſes, and meaſuring it upon your Line of equal parts, you ſhall find it to contain 135. And that is the length of the Perpendicular C A.

The Analogie or Proportion is,

As the Sine of the Angle at C is to the Log. of A B,
So is the Sine of the Angle B to the Logar. of C A.

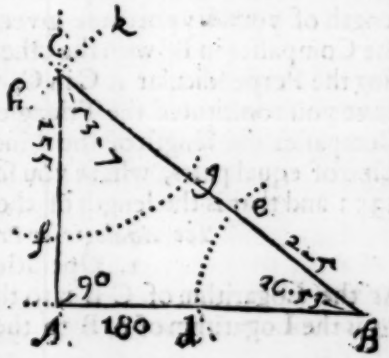
Or,

As the Radius is to the Logar. of A B,
So is the Tangent of B to the Logar. of C A.

CASE IV.

The Hypotenuſe C B 225, the Angle C 53 degr. 7 min. and the Angle at B 36 degr. 53 min. given, to finde the Baſe B A, and the Perpendicular C A.

DRaw a right Line C B containing 225 of your Line of equal parts; then taking 60 deg. out of your Line of Chords, ſet one foot of the Compaſſes in B, and with the other deſcribe the Arch *e d*; alſo (the Compaſſes continuing at the ſame diſtance) place one foot in C, and with the other deſcribe the



Arch *g f*. Then from the Point B, and through the Point *d*, draw a right Line; alſo from the Point C, and through the Point *f*, draw another right Line: theſe two Lines will interſect or croſs each other in the Point A, forming the Triangle C A B. Laſtly, take the Line A B in your Compaſſes, and applying it to your Scale of equal parts, you ſhall finde it to contain 180; and that is the length of the Baſe A B. Likewiſe A C being taken in the Compaſſes, and meaſured upon the Line of equal parts, will be found to contain 135, which is the length of the Perpendicular C A.

The Analogie or Proportion is,

As the Radius is to the Logarithm of C B,
So is the Sine of C to the Logarithm of A B,
And the Sine of B to the Logarithm of C A.

CASE V.

The Hypotenuſe C B 225, *and the Baſe A B* 180, *being given, to finde the Perpendicular C A.*

DRaw a right Line A B containing 180 of your Scale of equal parts, and upon the end A erect a Perpendicular A C. Then take out of your Scale of equal parts 225, (the length of your Hypotenuſe given,) and ſetting one foot of the Compaſſes in B, with the other deſcribe the Arch *h k*, cutting the Perpendicular A C in C, then draw the Line C B: ſo have you conſtituted the Triangle C A B. Laſtly, take in your Compaſſes the length of the Line A C, and apply it to your Line of equal parts, where you ſhall finde that it will contain 135: and that is the length of the Perpendicular C A.

The Analogie or Proportion is,

1. Operation.

As the Logarithm of C B is to the Radius,
So is the Logarithm of A B to the Sine of C.

2. Opera-

2. Operation.

As the Radius is to the Logarithm of C B,
So is the Sine of B (the Complement of C) to the Log. of C A.

Case VI.

The Baſe A B 180, *the Angle C* 53 *degr.* 7 *min. and the Angle B* 36 *degr.* 53 *min. being given, to finde the Hypotenuſe C B.*

DRaw a right Line A B containing 180 parts of your Scale, for the Baſe of your Triangle, and on the end A erect a Perpendicular A C. Then take 60 degr. out of your Line of Chords, and upon the Point B, with that diſtance, deſcribe the Arch *d e*; and (becauſe the Angle at B is 36 degr. 53 min.) take 36 degr. 53 min. from your Line of Chords, and ſet it upon the Arch from *d* to *e*. Then from B, through the Point *e*, draw a right Line, till it meet with the Perpendicular before drawn, which it will do in the Point C. And thus have you protracted your Triangle C A B. Laſtly, take in your Compaſſes the length of the Hypotenuſe C B, and meaſure it upon your Scale of equal parts, and you ſhall finde it to contain 225.

The Analogie or Proportion is,

As the Sine of C is to the Logarithm of A B,
So is the Radius to the Logarithm of C B.

Case VII.

The Baſe A B 180, *and the Perpendicular C A* 135, *being given, to finde the Hypotenuſe C A.*

DRaw a right Line A B containing 180 equal parts, and upon the end A erect the Perpendicular A C, and out of your Scale of equal parts take the length thereof 135, which ſet from A to C, and draw the Line C B, which con-

ſtitutes the Triangle C A B. Laſtly, take the length of the Hypotenuſe C B in your Compaſſes, and meaſuring it upon your Line of equal parts, you ſhall finde it to contain 225.

The Analogie or Proportion is,

1. Operation.

As the Logarithm of A B is to the Logarithm of C A,
So is the Radius to the Tangent of B.

2. Operation.

As the Sine of B is to the Logarithm of C A,
So is the Radius to the Logarithm of C B.

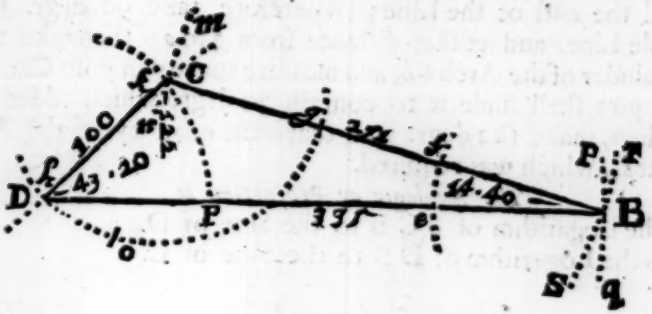
Theſe are the ſeverall Varieties or Caſes that can at any time fall out in the Solution of Right-angled plain Triangles, wherefore we will now proceed to the Solution of Oblique plain Triangles.

II. *Of Oblique-angled plain Triangles.*

THE *Triangle* which I ſhall make uſe of in the Solution of the ſeverall *Caſes* appertaining to an *Oblique-angled plain Triangle* ſhall be this following, C D B, in which

		parts
D B, the Baſe,	contains	335
C B, the longer Side,		271
D C, the ſhorter Side,		100
And		deg. m.
C, the obtuſe Angle,	contain	122—00
D, B, the 2 acute Angles,		43—20
		14—40

CASE



Case I.

Two Sides, as the Base D B 335, and the Side C B 271, and the Angle D 43 degr. 20 min. opposite to C B, to finde the Angle at C, opposite to the Base D B.

DRaw a right Line D B representing the Base of your Triangle, which, by help of your Scale of equal parts, make to contain 335. Then upon the Point D, with the distance of 60 degr. of your Line of Chords, describe the Arch *k l*, and from your Chords take 43 degr. 20 min. the quantity of the Angle at D, and set it upon the Arch-line from *l* to *k*, drawing the Line C D. And because your other given Side B C contains 271 parts, take 271 out of your Line of equal parts, and setting one foot in B, with the other describe the Arch *m n*, crossing the former Arch *k l* in the Point C: then draw the Line C B. So shall you have constituted the Triangle C D B. Lastly, because it is the Angle at C that is required, take 60 degr. of your Chords, and upon C describe the Arch *g h*, and taking the distance between *g* and *h*, apply it to your Line of Chords,

and

and you ſhall finde it to reach from the beginning thereof beyond the end of the Line; wherefore take 90 degr. the whole Line, and ſet that diſtance from *g* to *o*; then take the remainder of the Arch *o h*, and meaſure that upon your Chord, and you ſhall finde it to contain 32 degr. which added to 90 degr. make 122 degr. and that is the quantity of the Angle at C, which was required.

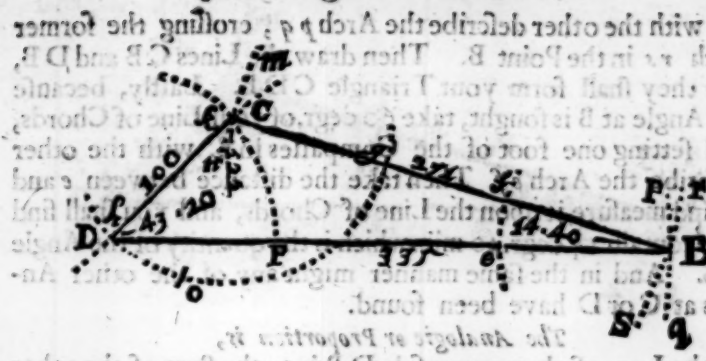
The Analogie or Proportion is,
As the Logarithm of B C is to the Sine of D,
So is the Logarithm of D B to the Sine of C.

Case II.

The Baſe D B 335, *and the Side D C* 100, *with the Angle D*, 43 *degr.* 20 *min. contained between them, to finde either of the other Angles at B and C.*

DRaw a right Line, as D B, containing 335 of your Scale of equal parts, which ſhall be the Baſe of your Triangle. Then with 60 degr. of your Line of Chords, upon the Point D deſcribe the Arch *k l*; and becauſe the given Angle at D contains 43 degr. 20 min. take 43 degr. 20 min. from your Line of Chords, and ſet it from *l* to *k*, drawing the Line D *k*. Again, becauſe the given Side D C contains 100, ſet 100 of your Line of equal parts from D to C; then drawing a right Line from C to B, you ſhall by that means find the oblique-angled Triangle C D B. Laſtly, being the other two Angles at B and C are to be found, with 60 degr. of your Chord on the Point B deſcribe the Arch *e f*; alſo upon the Point C deſcribe the Arch *g o h*. Then if you take the diſtance between *e* and *f* in your Compaſſes, and meaſure it upon your Line of Chords, you ſhall finde it to contain 14 degr. 40 min. And that is the quantity of the Angle at B. Then being the Angle at C, which is alſo required, is obtuſe, and contains above 90 degr. take 90 degr. out of your Line of Chords,

and



and set that distance upon the Arch *g h*, from *g* to *o*; and taking the other part of the Arch *o h* in your Compasses, measure that upon your Chord, and you shall find it to contain 32 degr. which added to 90 degr. makes in all 122 degr. And such is the quantity of the other enquired Angle at C.

The Analogie or Proportion is,

As the Log. of the Sum of the two Sides given, C D and C B, is to the difference of those Sides,

So is the Tang. of half the Sum of the two unknown Angles, C and B, to the Tangent of half their difference.

CASE III.

The three Sides, D B 335, C B 271, and D C 100, being given, to finde any of the Angles, as B.

DRaw a right Line C D, containing 100 of your Line of equal parts. Then the other side of the Triangle being 271, take 271 out of your Scale of equal parts, and setting one foot of the Compasses in C, with the other describe the Arch *e f*. Also take the length of your Base 335 out of your Line of equal parts, and setting one foot of the Compasses in

D, with the other describe the Arch *p q*, crossing the former Arch *r s* in the Point B. Then draw the Lines C B and D B, and they shall form your Triangle C D B. Lastly, because the Angle at B is sought, take 60 degr. of your Line of Chords, and setting one foot of the Compasses in B, with the other describe the Arch *e f*. Then take the distance between *e* and *f*, and measure it upon the Line of Chords, and you shall find it to contain 14 degr. 40 min. which is the quantity of the Angle at B. And in the same manner might any of the other Angles at C or D have been found.

The Analogie or Proportion is,

As the Log. of the greater side D B is to the Sum of the other two Sides, D C and C B,

So is the difference of the two Sides to a fourth Sum.

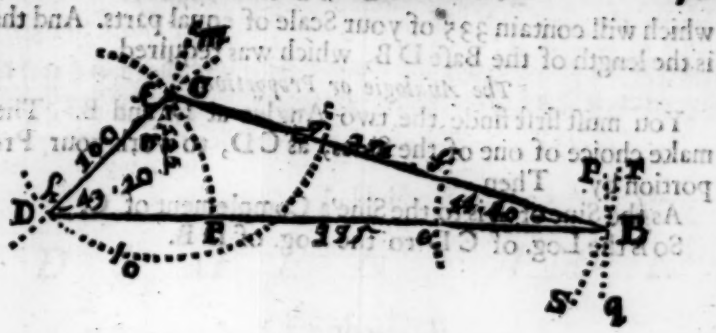
Which fourth Sum being taken from the Base, will leave another number, the half whereof will be the place in the Base where a Perpendicular let fall from the obtuse Angle would fall upon the Base: and so the oblique Triangle is reduced into two right-angled, and may be resolved by the Precepts of right-angled Triangles.

Case IV.

The three Angles, C 122 degr. D 43 degr. 20 min. and B 14 deg. 40 min. being given, to finde any of the Sides, as B C.

IN this Case, where the three Angles are given, and none of the Sides, you are to take notice, that the Sides cannot be absolutely found themselves, but the Proportionality of them. Wherefore

Draw a Line, as D B, of any length, and taking 60 degr. of your Line of Chords, set one foot of the Compasses upon D, and with the other describe the Arch *l k*; also set one foot in B, and with the other describe the Arch *e f*. Then, because the Angle at D contains 43 degr. 20 min. take 43 degr. 20 min.



20 min. from your Line of Chords, and ſet them from *l* to *k*. Alſo the Angle at B being 14 degr. 40 min. take them likewiſe from your Line of Chords, and ſet them from *e* to *f*. This done, draw the Lines B *f* and D *k*, extending them till they meet one with another, which they will doe in the Point C. So have you conſtituted the Triangle C D B, the Sides whereof will be in proportion the one to the other as the Sides of this Triangle are.

Case V.

The two Sides D C 100, *and C B* 271, *with the Angle at C* 122 *degr. being given, to finde the Baſe D B.*

DRaw a right Line, as C B, containing 271 of your Line of equal parts, and on the Point C, with 60 degr. of your Line of Chords, deſcribe the Arch *g o h*. Then, becauſe the given Angle C contains 122 degr. it being 32 degr. above 90 degr. firſt take 90 degr. from off your Line of Chords, and ſet it upon the Arch from *g* to *o* 5 and then take 32 degr. from your Chord alſo, and ſet them upon the ſame Arch from *o* to *h*, and draw the Line C *h*: then take 100, the length of the other given Side, and ſet it from C to D, and draw the Line D B,

which will contain 335 of your Scale of equal parts. And that is the length of the Base D B, which was required.

The Analogie or Proportion.

You must first finde the two Angles at D and B. Then make choice of one of the Sides, as C D, to work your Proportion by. Then

As the Sine of B is to the Sine's Complement of C,
So is the Log. of C D to the Log. of D B.

The End of the Doctrine of the Dimension of right-lined Triangles.

THE

THE DIMENSION Of Spherical TRIANGLES, By A LINE OF CHORDS.

The Third EXERCISE.

AS *the* Sides *of* Right-lined Triangles *were measured by* Equal parts *and their* Angles *by a* Line of Chords; *so the* Sides *of* Spherical Triangles *being Arches of* great Circles *of the* Sphere, *both they and the* Angles *also may be measured by Chords onely.*

Of Spherical Triangles *there be two kinds, as there were of* Right-lined; *namely,* right-angled *and* oblique-angled: *of both which there are* 28 Cases, *viz.* 16 *of* right-angled, *and* 12 *of* oblique-angled. *And to*

observe the same Method in these *as I did in* plain Triangles, *I will begin with the Solution*

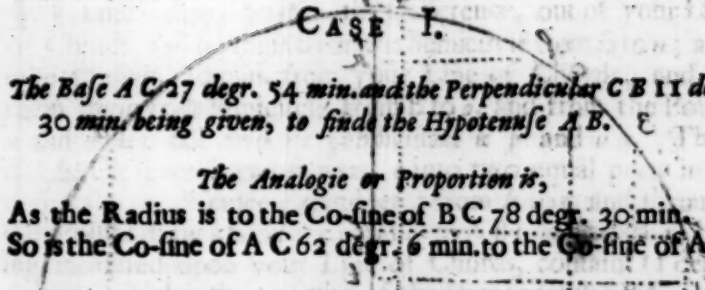
I. *Of Right-angled Spherical Triangles.*

THE *Right-angled spherical Triangle* which I shall make use of for my *Examples*, shall be this following, A B C, whose Sides and Angles contain as followeth.

			deg.	m.
The Side	A B	containeth	30	00
	B C		11	30
	A C		27	54
The Angle at	A	containeth	23	30
	B		69	22
	C		90	00

CASE I.

The Base A C 27 degr. 54 min. and the Perpendicular C B 11 degr. 30 min. being given, to find the Hypotenuse A B.



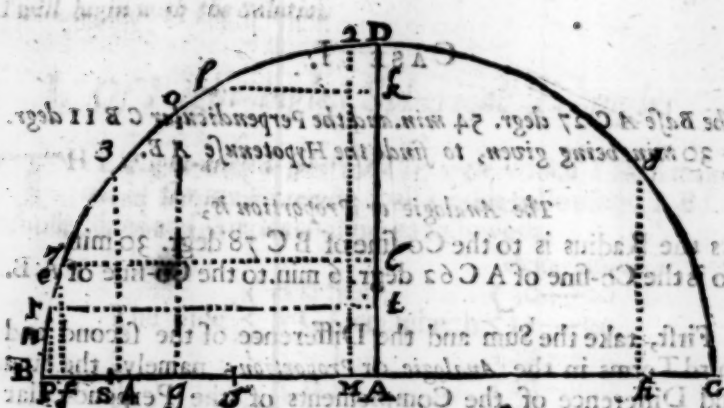
The Analogie or Proportion is,

As the Radius is to the Co-sine of B C 78 degr. 30 min.
So is the Co-sine of A C 62 degr. 6 min. to the Co-sine of A B.

First, take the Sum and the Difference of the second and third Terms in the *Analogie* or *Proportion*; namely, the Sum and Difference of the Complements of the Perpendicular B C 11 degr. 30 min. and the Base A C 27 degr. 34 min.

	degr.	m.		degr.	m.
The Base A C is	27	54.	its Comp.	62	06
The Perpendicular B C is	11	30.	its Comp.	78	30
			Sum	140	36
			Differ.	16	24

HAving found this Sum and Difference, draw a right Line B A C; then take 60 degr. out of your Line of Chords, and setting one foot of the Compasses in A, with the other describe the Semicircle B D G, and upon the Centre A erect the Perpendicular A D. 1 This done, take 16 degr. 24 min. (the Difference) out of your Line of Chords, and set them from B to *e*; also take the Sum 140 degr. 36 min. (or rather 39 degr. 24 min. the Complement thereof to 180 degr.) and set them from C to *g*, and from the Points *e* and *g* let fall the two Perpendiculars *e f* and *g b*: then divide the space between *f* and *b* into two equal parts in M, and set the distance



Mh or M *f* from A to *k*, and draw the Line *k l* parallel to A B, cutting the Semicircle in *l*: ſo ſhall D *l* be the quantity of the Hypotenuſe A B, which if you meaſure upon your Line of Chords, you will finde to contain 30 degr.

Case II.

The Hypotenuſe A B 30 degr. and the Angle at the Baſe A 23 degr. 30 min. being given, to find the Perpendicular B C.

The Analogie or Proportion is,

As the Radius is to the Sine of the Hypotenuſe A B 30 degr. So is the Sine of the Angle at A 23 degr. 30 min. to the Sine of the Perpendicular B C, 11 degr. 30 min.

Take the Sum and Difference of the Angle at A 23 degr. 30 min. and the Hypotenuſe A B 30 degr.

The Sum is degr.

The Difference is

Having

HAving drawn your Semicircle B D C, as in the laſt Caſe, take 6 degr. 30 min. the Difference, out of your Line of Chords, and ſet them upon the Semicircle from B to *n*; alſo take 53 degr. 30 min. from your Line of Chords, and ſet them upon your Semicircle from B to *o*, and from the Points *n* and *o* let fall two Perpendiculars *n p* and *o q*. Then divide the ſpace between *p* and *q* into two equal parts in *s*; then take the diſtance *q s*, and ſet it from A to *t*, and through the Point *t* draw a Line *t r* parallel to A B: ſo ſhall B *r*, being meaſured upon your Line of Chords, contain 11 degr. 30 min. which is the quantity of the Perpendicular B C.

Case III.

The Baſe A C 27 degr. 54 min. and the Angle at the Baſe A 23 d. 30 min. being given, to find the Angle at the Perpendicular B.

The Analogie or Proportion is,

As the Radius is to the Co-ſine of A C 62 degr. 6 min.
So is the Sine of the Angle at A 23 degr. 30 min. to the Co-ſine of the Angle at B.

Take the Sum and Difference of the Co-ſine of A C 62 d. 6 min. and of the Sine of the Angle at A 23 degr. 30 min.

	degr.	m.
The Sum is	85	36
The Difference is	38	36

BEing thus far prepared, and having drawn your Semicircle B D C, out of your Line of Chords take 85 degr. 36 m. and ſet them upon your Semicircle from B to 2; alſo take the Difference 38 degr. 36 min. from your Chord, and ſet them from B to 3, and let fall the two Perpendiculars, 3 4, and 2 M. Then divide the diſtance between M and 4 into two

equal parts in the Point 5, and taking the distance M 5 in your Compasses, set it upon the Line A D, from A to 6. Lastly, draw the Line 6 7. parallel to B A, so shall D 7 being measured upon your Line of Chords contain 69 degr. 22 min. the quantity of the Angle at B required.

These three Cases are all (in Sines alone) that have the Radius in the first place of the Analogie or Proportion, and so consequently all that can be wrought by this Artifice: wherefore those which follow must be resolved by other means.

Case IV.

The Perpendicular B C 11 degr. 30 min. and the Angle at the Base A 23 degr. 30 min. to finde the Hypotenuse A B.

The Analogie or Proportion is,

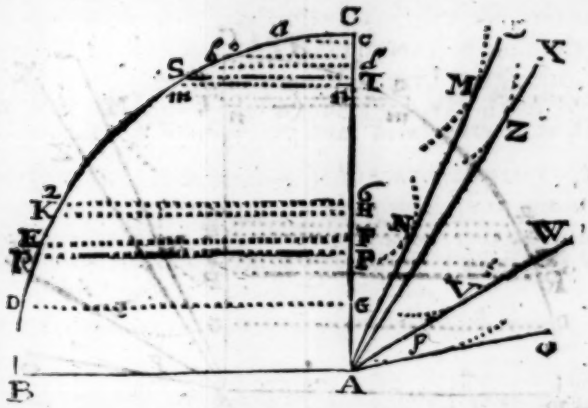
As the Sine of the Angle at A 23 degr. 30 min. is to the Sine of the Side B C 11 degr. 30 min.
So is the Radius to the Sine of A B.

FIrst, draw a right Line B A, and upon the Point A, with 60 degr. of your Line of Chords, describe the Quadrant A B C.

Then take 23 degr. 30 min. the quantity of the given Angle at A, out of your Line of Chords, and set them upon the Quadrant from B to E. Also take 11 degr. 30 min. the quantity of the given Side B C, out of your Line of Chords, and set them from B to D; and draw the Lines E F and D G both parallel to B A.

Then with your Compasses take the distance between A and F, and setting one foot in C, with the other describe the Arch M, and from A draw the Line A L so that it may onely touch the Arch M. Then taking the distance A G in your Compasses, set one foot of them upon the Line A C, and draw it

gently



gently and softly along the Line A C, till the other foot, being turned about, will onely just touch the Line A L; and when it so toucheth, mark where the other Point resteth upon the Line A C, which you shall finde it to doe at the Point 6. Lastly, through the Point 6 draw the Line 6 2, parallel to A B. So shall the distance 2 B 6, being measured upon the Line of Chords, contain 30 degr. the quantity of the Hypotenuse A B, which was the Side required.

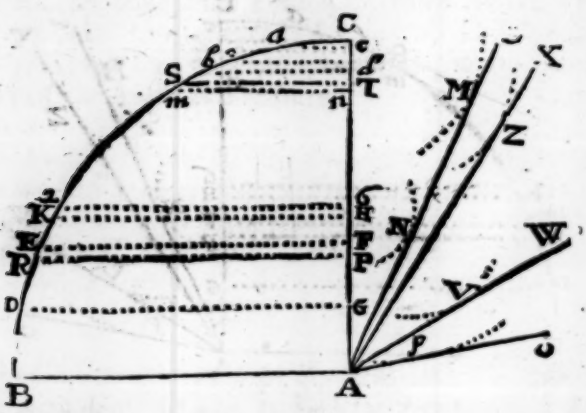
Case V.

The Perpendicular B C 11 degr. 30 min. and the Angle at the Base A 23 degr. 30 min. being given, to finde the Angle at the Perpendicular B.

The Analogie or Proportion is,

As the Co-sine of B C 78 degr. 30 min. is to the Co-sine of A 66 degr. 30 min.

So is the Radius to the Sine of the Angle at the Perpendicular B.



HAving drawn your Quadrant A B C, take from your Line of Chords 78 degr. 30 min. the Complement of the given Side B C, and ſet it from B to *a*. Alſo take from your Chords 66 degr. 30 min. the Complement of the given Angle at A, and ſet them from B to *b*, and through the Points *a* and *b* draw the Lines *a c* and *b d* parallel to B A. Then taking in your Compaſſes the diſtance A *c*, ſet one foot in C, and with the other foot deſcribe the Arch *p*, and draw the Line A O ſo that it onely touch the Arch *p*. Likewiſe, take in your Compaſſes the diſtance A *d*, and with that diſtance, ſetting one foot upon the Line A C, move it gently along that Line till the other foot, being turned about, do onely touch the Line A O. So ſhall you finde the point of the Compaſſes to reſt in the Point *e*, through which Point draw the Line *e o* parallel to B A. Then meaſure the diſtance B *o* upon your Line of Chords, and you ſhall finde it to contain 69 degr. 22 min. the quantity of the enquired Angle at B.

CASE

CASE VI.

The Angle at the Base A 23 degr. 30 min. and the Angle at the Perpendicular B 69 degr. 22 min. being given, to finde the Base A C.

The Analogie or Proportion is,

As the Sine of the Angle at A 23 degr. 30 min. is to the Co-sine of the Angle at B 20 degr. 38 min.
So is the Radius to the Co-sine of the Base A C.

YOur Quadrant being drawn, take 23 degr. 30 min. the quantity of the Angle at A, and set it from B to E. Also take 20 degr. 38 min. the Complement of the Angle at B, out of your Line of Chords, and set them upon your Quadrant from B to R, and through the Points E and R draw the Lines E F and R P parallel to A B.

Then take in your Compasses the distance A F, and setting one foot in C, with the other describe the Arch M, and draw the Line A L so that it onely touch the Arch M. Then in your Compasses take the distance A P, and setting one foot upon the Line A C, move it along till the other, being turned about, do onely touch the Line A L; and when it so toucheth, note upon what part of the Line A C the Compass-point resteth, which you will finde it to doe at the Point T; through that Point draw the Line T S parallel to B A. So shall the distance B S, being measured upon your Line of Chords, contain 62 d. 6 min. the Complement of the enquired Side: or, S C will give 27 degr. 54 min. the Side it self.

Case VII.

The Base A C 27 degr. 54 min. and the Hypotenuse A B 30 degr. being given, to finde the Angle at the Perpendicular B.

The Analogie or Proportion is,

As the Sine of the Hypotenuse A B 30 degr. is to the Radius, So is the Sine of the Base A C 27 degr. 54 min. to the Sine of the Angle at the Perpendicular B.

HAving drawn your Quadrant, take from your Line of Chords 30 degr. the quantity of the Hypotenuse, and set them upon your Quadrant from B to 2 3 also take 27 degr. 54 min. the quantity of the Base, from your Chords, and set them from B to K, and draw the Lines 2 6 and K H, both parallel to the Line B A. Then taking in your Compasses the distance A 6, set one foot of them in C, and with the other describe the Arch Z, and draw the Line A X, so that it onely touch the Arch Z. Then taking in your Compasses the distance A H, set one foot upon the Line A C, moving it along till the other foot, being turned about, will onely touch the Line A Z; and where the point of the Compasses resteth upon the Line A C, which it will doe at *e*, through that Point draw the Line *o e* parallel to B A. So shall B *o*, being measured upon your Line of Chords, give you 69 degr. 22 min. the quantity of the enquired Angle at B.

Case VIII.

The Base A C 27 degr. 54 min. and the Hypotenuse A B 30 degr. being given, to finde the Perpendicular B C.

The Analogie or Proportion is,

As the Co-sine of the Base A C 62 degr. 6 min. is to the Radius, So is the Co-sine of the Hypotenuse 60 degr. to the Co-sine of the Perpendicular B C.

Having

HAving drawn your Quadrant A B C, take out of your Line of Chords 62 degr. 6 min. the Co-sine of the Base A C, and set them from B to S. Also take from the Chords 60 degr. the Co-sine of the Hypotenuse, and set them from B to *m*: and draw the Lines S T and *m n* both parallel to B A. Then taking the distance A T in your Compasses, set one foot in C, and with the other describe the Arch V, and draw the Line A W so that it may onely touch the Arch V. Then taking A *n* in your Compasses, move one foot thereof gently along the Line C A, till the other, being turned about, doth onely touch the Line A W; and where the Point resteth upon the Line C A, which you will finde to be at *c*, there make a mark, and draw the Line *c a* parallel to B A. Lastly, take the distance from B to *a*; and measure it upon your Line of Chords, where you shall finde it to contain 78 degr. 30 min. the Complement of the Perpendicular; or, C A measured upon the Chord will give you 11 d. 30 min. the Perpendicular it self.

These Five last are all the Cases in a Right-angled Spherical Triangle that are resolvable by Sines alone. Those which follow are to be resolved by Sines and Tangents joyntly, and so will require another manner of Operation then the former.

CASE IX.

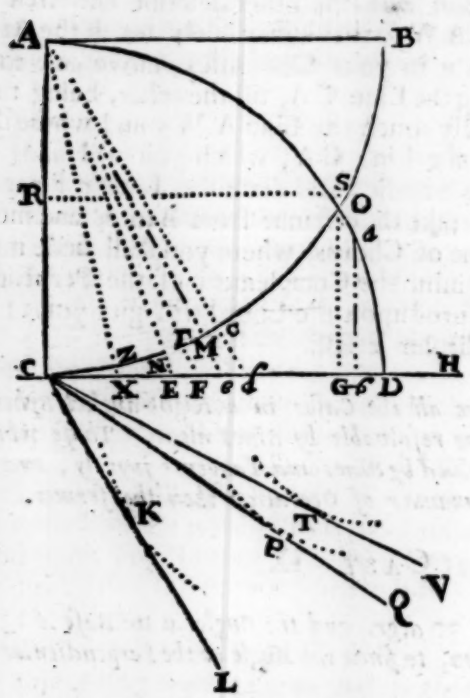
The Hypotenuse A B 30 *degr. and the Angle at the Base A* 23 *d.* 30 *min. being given, to finde the Angle at the Perpendicular B.*

The Analogie or Proportion is,

As the Radius is to the Co-sine of the Hypotenuse A B 60 deg. So is the Tangent of the Angle at the Base A 23 degr. 30 min. to the Co-tangent of the Angle at the Perpendicular B.

First,

First, draw a right Line, as C H, and upon one end thereof (as at C) erect the Perpendicular C A, and with the distance of 60 degr. of your Line of Chords, upon the Centre C describe the Quadrant C A D. Also upon the Point A with 60 degr. of your Chord describe the Quadrant A B C.



Being thus prepared, First, take 60 deg. the Co-sine of the Hypotenuse A B, and set them from A to O; also take 23 d. 30 m. the quantity of the Angle at A, and set them from C to r; and draw the Line A M F, and the Line O G, parallel to A C.

Then take in your Compasses the distance between C and G, and setting one foot in D, with the other describe the Arch K, and draw the Line A L so that it onely touch the Arch K. Then placing one foot of the Compasses in F, take the least distance you can to the Line C L, which set from C to E. Lastly, draw the Line A E, cutting the Quadrant C B in N. So the distance

C N

C N measured upon your Chords shall give you 20 degr. 38 min. the Complement of the Angle at B, which was required; or, the distance B N will give you 69 degr. 22 min. the Angle it self.

CASE X.

The Hypotenuse A B 30 degr. and the Angle at the Base A 23 d. 30 min. being given, to finde the Base A C.

The Analogie or Proportion is,

As the Radius is to the Co-sine of the Angle at A 66 d. 30 m. So is the Tangent of A B the Hypotenuse 30 degr. to the Tangent of the Base A C.

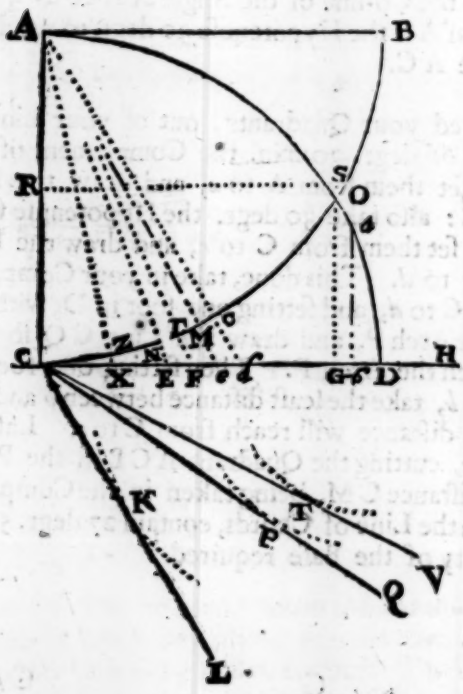
HAving prepared your Quadrants, out of your Line of Chords take 66 degr. 30 min. the Complement of the Angle at A, and set them from A to *a*, and draw the Line *a b* parallel to A C: also take 30 degr. the Hypotenuse from your Chord, and set them from C to *c*, and draw the Line A *c*, prolonging it to *d*. This done, take in your Compasses the distance from C to *d*, and setting one foot in D, with the other describe the Arch P, and draw the Line C Q so that it may onely touch the Arch P. Then setting one foot of the Compasses in *b*, take the least distance between *b* and the Line C Q, which distance will reach from C to *e*. Lastly, draw the Line A *e*, cutting the Quadrant A C B in the Point M. So shall the distance C M, being taken in the Compasses and measured upon the Line of Chords, contain 27 degr. 54 m. which is the quantity of the Base required.

Case XI.

The Base A C 27 degr. 54 min. and the Angle at the Base A 23 degr. 30 min. being given, to finde the Perpendicular B C.

The Analogie or Proportion is,

As the Radius is to the Sine of the Base A C 27 degr. 54 min. So is the Tangent of the Angle at A 23 degr. 30 min. to the Tangent of the Perpendicular B C.



YOur Quadrants being prepared, out of your Line of Chords take 27 d. 54 m. the quantity of the given Base A C, and set them from A to S, drawing the Line S R parallel to C D. Also take out of your Chords 23 degr. 30 minutes, the quantity of the given Angle at A, and set them from C to r, and draw the Line A r, prolonging it to F.

This done, take

take in your Compasses the distance A R, and setting one foot in D, with the other describe the Arch T, and by the side thereof draw the Line C V onely to touch it. Then set one foot of your Compasses in F, and with the other take the neerest distance to the Line C V, which distance will reach from C to X. Lastly, draw the Line A X, which will cut the Quadrant A B C in the Point Z. So shall C Z, being measured upon the Line of Chords, contain 11 degr. 30 min. which is the quantity of the Perpendicular B C, which was required.

Case XII.

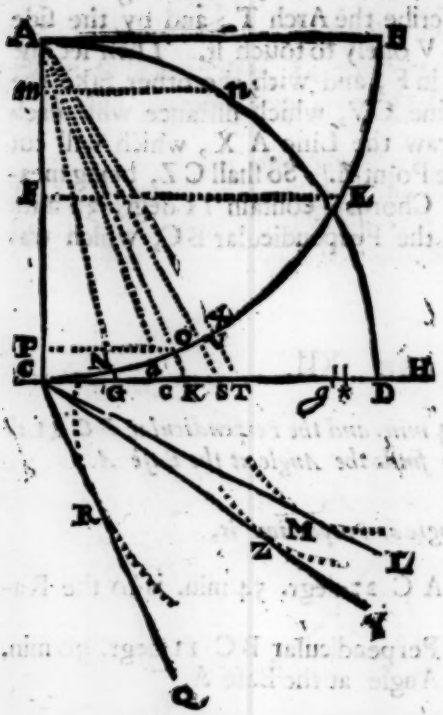
The Base A C 27 degr. 54 min. and the Perpendicular B C 11 d. 30 min. being given, to finde the Angle at the Base A.

The Analogie or Proportion is,

As the Sine of the Base A C 27 degr. 54 min. is to the Radius,
So is the Tangent of the Perpendicular B C 11 degr. 30 min. to the Tangent of the Angle at the Base A.

DRaw the two Quadrants A B C and C A D, as before. Then take out of your Line of Chords 27 degr. 54 m. the quantity of the given Base A C, and set them upon the Quadrant from B to E, and draw the Line E F parallel to C D. Also take 11 degr. 30 min. the quantity of the Perpendicular B C, and set them upon the Quadrant from C to N, and draw the Line A N, cutting the Line C H in G.

This done, take in your Compasses the distance A F, and setting one foot in D, with the other describe the Arch M, and close by the out-side of it draw the Line C L. Then



take in your Compasses the distance C G, and setting one foot of them upon the Line C D, move it along till the other foot, being turned about, will onely touch the Line C L; and when it so toucheth, mark where the other foot resteth upon the Line C D, which it will do at K. Lastly, draw the Line A K, cutting the Quadrant C B in the Point O. So shall C O, being measured on the Line of Chords, contain 23 degr. 30 min. the quantity of the enquired Angle at A.

CASE XIII.

The Base A C 27 degr. 54 min. and the Angle at the Base A 23 d. 30 min. being given, to finde the Hypotenuse A B.

The Analogie or Proportion is,

As the Co-sine of the Angle at A 66 d. 30 m. is to the Radius, So is the Tangent of the Base A C 27 degr. 54 min. to the Tangent of the Hypotenuse A B.

The

THE Quadrants being drawn, take the quantity of the given Base A C 27 degr. 54 min. out of the Line of Chords, and set them upon the Quadrant from C to V, and draw the Line A V, continuing it till it cut the Line C D in S. Also take 66 degr. 30 min. the Complement of the given Angle at A, out of the Chords, and set them from B to O, and draw the Line O P parallel to C D or A B.

This done, take the distance A P in your Compasses, and setting one foot in D, with the other describe the Arch R, drawing the Line C Q onely to touch it. Then take in your Compasses the distance C S, and placing one foot of them upon the Line C D, move it along the same, till the other, being turned about, will onely touch the Line C Q; and where the Compas-point resteth upon the Line C D, (which you will finde it to doe at T,) there make a mark, and from it draw the Line A T, cutting the Quadrant A B C in the Point X. So shall C X, being measured on the Line of Chords, give you 30 degr. the quantity of the Hypotenuse required.

CASE XIV.

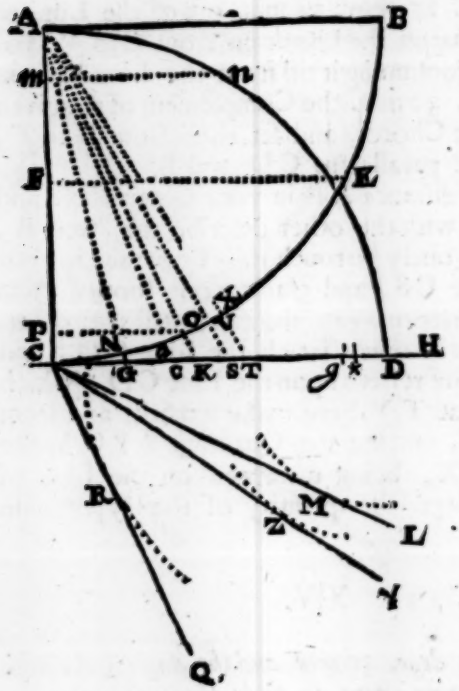
The Perpendicular B C 11 *degr.* 30 *min. and the Angle at the Base A* 23 *degr.* 30 *min. being given, to finde the Base A C.*

The Analogie or Proportion is,

As the Tangent of the Angle at A 23 degr. 30 min. is to the Tangent of the Perpendicular B C 11 degr. 30 min. So is the Radius to the Sine of the Base A C.

HAving drawn your Quadrants, take out of your Line of Chords 23 degr. 30 min. the quantity of the Angle at A, and set them from C to O, and draw the Line A O, continuing it till it cut the Line C D in K. Also take out of your Line

of Chords 11 degr. 30 min. the quantity of the Perpendicular given, and ſet them from C to N, drawing the Line A N, and prolonging it to G.



This done, take in your Compaſſes the diſtance between C and K, and ſetting one foot in D, with the other deſcribe the Arch M, and draw the Line C L. Then take in your Compaſſes the diſtance C G, and ſetting one foot upon the Line C D, move it gently along the ſame, till the other, being turned about, do only touch the Line C L; and when it ſo toucheth, keeping the Compaſſes at the ſame diſtance, ſet it from A to F. Laſtly, draw the Line F E parallel to A B. So ſhall the diſtance B E, being meaſured on your Line of Chords, contain 27 degr. 54 min. the quantity of the Baſe A C, which was required.

CASE

Case XV.

The Base A C 27 degr. 54 min. and the Hypotenuse A B 30 degr. being given, to finde the Angle at the Base A.

The Analogie or Proportion is,

As the Tangent of the Hypotenuse A B 30 degr. is to the Tangent of the Base A C 27 degr. 54 min.
So is the Radius to the Co-sine of the Angle at A.

THE Quadrants being drawn, out of your Line of Chords take 30 degr. the quantity of the Hypotenuse given, and set them upon the Quadrant from C to X. Also take 27 degr. 54 min. the quantity of the given Base, from the Line of Chords, and set them upon the same Quadrant from C to V, and draw the Lines A X and A V, prolonging them to T and S.

This done, take in your Compasses the distance between C and T, and setting one foot in D, with the other describe the Arch Z, and draw the Line C Y onely to touch it. Then take in your Compasses the distance C S, and placing one foot upon the Line C D, move it gently along, till the other, being turned about, do onely touch the Line C Y; and where the Point resteth upon the Line C D make a mark, which will be at *. Then take the distance C *, and set it from A to P, and draw the Line P O parallel to C D. So shall B O, being measured upon the Line of Chords, give you 66 degr. 30 min. the Complement of the Angle at A, which was required; or, the distance O C upon the Chord shall give you 23 degr. 30 min. the Angle it self.

Case

Case XVI.

The Angle B at the Perpendicular 69 degr. 22 min. and the Angle at the Base A 23 degr. 30 min. being given, to finde the Hypotenuse A B.

The Analogie or Proportion is,

As the Co-tangent of the Angle at B 20 degr. 38 min. is to the Tangent of the Angle at A 23 degr. 30 min.
So is the Radius to the Co-sine of the Hypotenuse A B.

HAving drawn the two Quadrants, as before, take out of your Line of Chords 23 degr. 30 min. the quantity of the Angle at A, and set them from C to O. Also take 20 degr. 38 min. from your Chord, (the Complement of the Angle at B) and set them from C to *a*, and draw the Lines A O and A *a*, continuing them to *c* and K upon the Line C D.

This done, take in your Compasses the distance C K, and setting one foot in D, with the other describe the Arch M, and draw the Line C L that it onely touch the Arch M. Also take in your Compasses the distance C *c*, and setting one foot upon the Line C D, move it gently along the same till the other, being turned about, do onely touch the Line C L; and where the Compass-point resteth upon the Line C D make a mark, as at *g*. Lastly, take the distance C *g*, and set it from C to *m*, and draw the Line *m n* parallel to A B. So shall the distance D *n*, measured on the Line of Chords, contain 60 d. the Complement of the Hypotenuse required; or, A *n* shall contain 30 degr. the Hypotenuse it self.

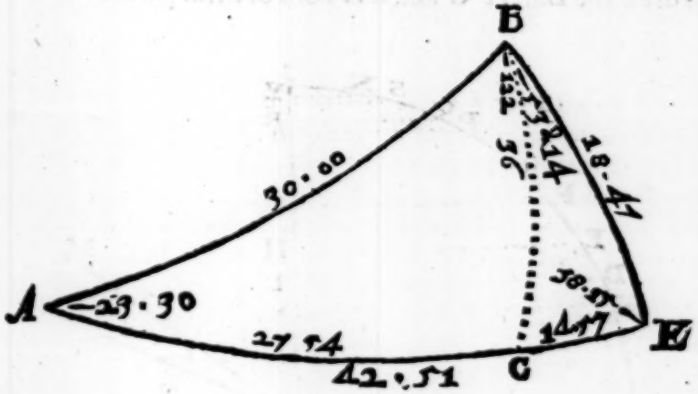
These 16 Cases are all that can be proposed in a Right-angled Spherical Triangle. There are 12 other Cases which belong to Oblique-angled Spherical Triangles, which we now come to resolve.

II. *Of*

II. *Of Oblique-angled Spherical Triangles.*

THE *Triangle* that I ſhall make uſe of in the Solution of the 12 Caſes of an *Oblique-angled Spherical Triangle* ſhall be this, A B E: whoſe *Sides* and *Angles* are as followeth.

			degr. m.
The Side	A B, the Baſe,	contains	30—00
	B E		18—47
	A E		42—51
The Angle	A	contains	23—30
	B		122—36
	E		38—15



H CASE

CASE I.

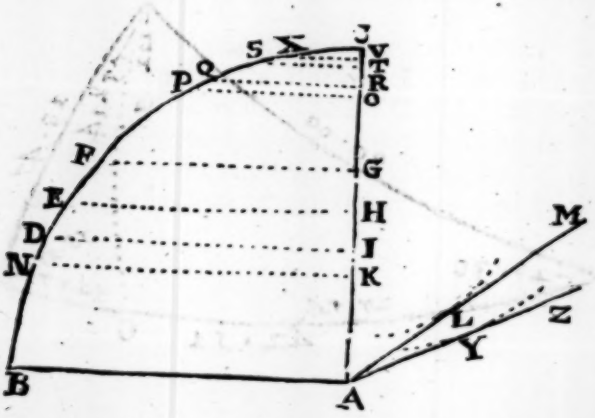
The Angle at E 38 degr. 15 min. the Angle at A 23 degr. 30 m. and the Side A B 30 degr. being given, to finde the Base B E.

The Analogie or Proportion is,

As the Sine of the Angle at E 38 degr. 15 min. is to the Side A B 30 degr.

So is the Sine of the Angle at A 23 degr. 30 min. to the Sine of the Side B E.

First, draw the Quadrant A B C: then out of your Line of Chords take 38 degr. 15 min. the quantity of the Angle at E, and set it upon the Quadrant from B to F. Also take 30 degr. the quantity of the given Side A B, out of your Line of Chords, and set them upon the Quadrant from B to E, and draw the Lines F G and E H both of them parallel to A B.



This done, take in your Compasses the distance A H, and setting

ſetting one foot of that extent in G, with the other deſcribe the Arch L, and draw the Line A M ſo that it may onely touch the Arch L. Then ſetting one foot of the Compaſſes in I, with the other take the neareſt diſtance to the Line A M: this diſtance ſet from A to K, and draw the Line N K parallel to A B. So ſhall the diſtance B N, meaſured upon the Line of Chords, contain 18 degr. 47 min. the quantity of the enquired Side B E.

Case II.

The Side A B 30 degr. the Angle at E 38 degr. 15 min. and the Side B E 18 degr. 47 m. being given, to finde the Angle at A.

The Analogie or Proportion is,

As the Sine of the Baſe A B 30 degr. is to the Sine of the Angle at E 38 degr. 15 min.
So is the Sine of the Side B E 18 degr. 47 min. to the Sine of the Angle at A.

YOur Quadrant being drawn, from your Line of Chords take 30 degr. the quantity of the Side A B, and ſet them from B to E. Alſo take 38 degr. 15 min. the quantity of the Angle at E, and ſet them from B to F. Likewiſe take 18 degr. 47 min. the quantity of the given Side B E, from your Line of Chords, and ſet them from B to N; and draw the Lines E H, and F G, and N K, all parallel to A B.

This done, take in your Compaſſes the diſtance A H, and ſetting one foot in G, with the other deſcribe the Arch L, drawing the Line A M onely to touch the Arch. Then take the diſtance A K in your Compaſſes, and ſetting one foot of them upon the Line A C, move it along that Line gently, till the other Point, being turned about, will onely touch the Line A M; and where the Compaſs-point reſteth upon the Line A C, which it will doe at the Point I, through I therefore draw

the Line I D parallel to B A. So shall B D, being measured on your Line of Chords, contain 23 degr. 30 min. the quantity of the enquired Angle at A.

Case III.

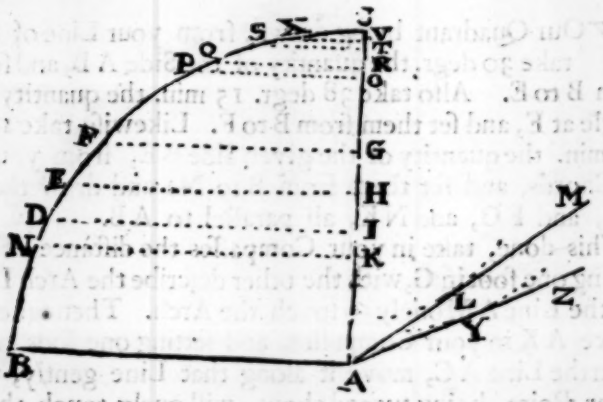
The Side A B 30 degr. and the Side B E 18 degr. 57 min. and the Angle at A 23 degr. 30 min. being given, to finde the Base A E.

The Analogie or Proportion is,

As the Co-sine of A B 60 degr. is to the Co-sine of B E 71 d. 13 min.

So is the Co-sine of A C 62 degr. 6 min. the place where a Perpendicular would fall from B, to the Co-sine of C E 75 degr. 3 min. Which C E, being added to A C, will give the quantity of the Side A E.

HAving drawn your Quadrant, take 60 degr. the Complement of A B, out of your Line of Chords, and set



them from B to P. Also take 71 degr. 13 m. the Complement of

of B E, and ſet them from B to S. In like manner take 62 d. 6 m. and ſet them from B to Q; and draw the Lines P O, S T, and Q R, all of them parallel to B A.

This done, take in your Compaſſes the diſtance A O, and ſetting one foot in T, with the other deſcribe the Arch Y; and draw the Line A Z onely to touch the Arch Y. Then take the diſtance A R, and ſetting one foot upon the Line A C, move it gently along the ſame, till the other foot, being turned about, do onely touch the Line A Z; and where the Compaſs-point reſteth, which it will doe at V, through that Point draw a Line V X parallel to B A. So ſhall the diſtance C X, meaſured upon your Line of Chords, contain 14 degr. 57 min. which added to the former Segment of the Side A C 27 degr. 54 min. the Sum will be 42 degr. 51 min. the quantity of the whole Side A E, which was required.

CASE IV.

The Angle at A 23 *degr.* 30 *min. the Angle at B* 122 *degr.* 36 *m. and the Side A B* 30 *degr. given, to finde the Angle at E.*

By the foregoing Caſes in Rectangled Spherical Triangles (if you let fall a Perpendicular from the Angle B upon the Side A E, as B C,) you may finde the Angle A B C 69 degr. 22 min. and the Angle C B E 53 degr. 14 min. which being obtained,

The Analogie or Proportion is,

As the Sine of the Angle A B C 69 degr. 22 min. is to the Sine of the Angle C B E 53 degr. 14 min.
So is the Co-ſine of A 66 degr. 30 min. to the Co-ſine of the Angle at E.

THE Angles being found and the Quadrant drawn, take 53 degr. 14 min. out of your Chord, and ſet them from B to E. Likewiſe take 69 degr. 22 min. and ſet them from B

to G. Also take 66 degr. 30 min. and set them from B to F; and draw the Lines G H, F I, and E K, all parallel to B A.

This done, take in your Compasses the distance between A and K, and setting one foot in H, with the other describe the Arch M, and draw the Line A N onely to touch the Arch. Again, setting one foot of the Compasses in I, with the other take the least distance to the Line A N, which distance set from A to L. Lastly, draw the Line L D parallel to B A, cutting the Quadrant in D. So shall the Arch B D, measured upon your Chords, contain 51 degr. 45 min. the Complement of the enquired Angle at E; or, D C shall give you 38 degr. 15 min. the Angle it self.

Case V.

The Angle at A 23 *degr.* 30 *min. the Angle at E* 38 *degr.* 15 *m. and the Side A B* 30 *degr. to finde the Angle A B E.*

By the preceding Cases of Right-angled Spherical Triangles, the Perpendicular B C being let fall, finde the Angle A B C, which will be found to be 69 degr. 22 min. This Angle being obtained,

The Analogie or Proportion is,

As the Co-sine of the Angle at A 66 degr. 30 min. is to the Co-sine of the Angle at E 51 degr. 45 min.
So is the Sine of A B C 69 degr. 22 min. to the Sine of the Angle C B E.

THE Angle A B C being found, and the Quadrant drawn, take 66 degr. 30 min. out of your Line of Chords, and set them from B to F. Also take 51 degr. 45 min. from the Chord, and set them from B to D. Likewise take 69 degr. 22 min. (the quantity of the Angle found A B C) from your Chord,

Chord, and ſet them from B to G, and draw the Lines F I, D L, and G H, all parallel to A B.

This done, take in your Compaſſes the diſtance A L, and ſetting one foot of that extent in I, with the other deſcribe the Arch M, and draw the Line A N onely to touch the Arch. Again, ſet one foot of the Compaſſes in H, and with the other take the leaſt diſtance to the Line A N, which had, it will reach from A to K; through which Point K draw the Line K E parallel to B A, cutting the Quadrant in E. So ſhall the diſtance B E, being meaſured on your Line of Chords, contain 53 degr. 14 min. the quantity of the remaining part of the Angle A B E; which being added to the Angle A B C (before found) 69 degr. 22 min. the Sum of them will be 122 degr. 36 min. the quantity of the Angle A B E, which was required.

Theſe Five laſt Caſes are all that in Oblique-angled Spherical Triangles are reſolvable by Sines onely, except the Three laſt, which we reſolve by another kind of Artifice. The 6. 7. 8. and 9. following have Tangents ingredient in the Proportion, and ſo do require a different way of reſolving, which is as followeth.

Case VI.

The Side A E 30 degr. the Baſe A E 42 degr. 51 min. and the Angle at A 23 degr. 30 min. being given, to find the Angle at E.

The Perpendicular B C being let fall from the Angle at B, you muſt find the Segment of the Baſe C E 14 degr. 57 min. by the preceding Caſes of Right-angled Spherical Triangles: Which had,

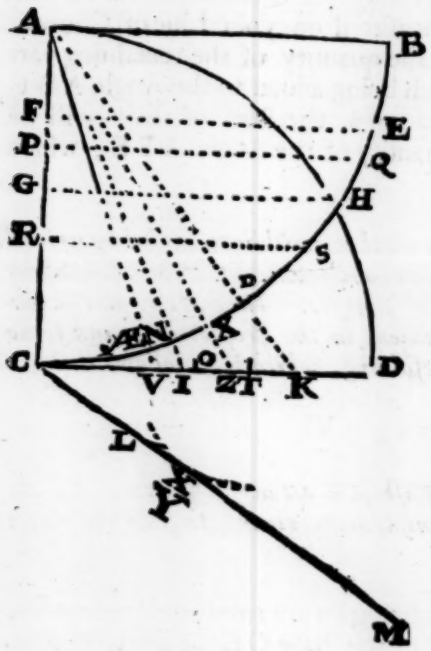
The Analogie or Proportion is,

As the Sine of the Side C E found 14 degr. 57 min. is to the Sine of A C 27 degr. 54 min.

So

So is the Tangent of the Angle at A 23 degr. 30 min. to the Tangent of the Angle at E.

YOur Quadrants A B C and C A D being drawn, take 14 degr. 57 min. the quantity of C E found, out of your Line of Chords, and ſet them from B to E. Alſo take 27 d. 54 min. the Segment of the Baſe A E, and ſet them from B to H; and draw the Lines E F and G H both parallel to A B. Alſo, take out of your Line of Chords 23 d. 30 min. the quantity of the Angle at A, and ſet them from C to N; and draw the Line A N, continuing it to I.



This done, take the diſtance A G, and ſet it from C to O. Alſo take the diſtance A F, and ſetting one foot in O, with the other deſcribe the Arch L; and draw the Line C M only that it may touch the Arch L. Then take in your Compaſſes the diſtance C I, ſet one foot upon the Line C D, moving it along the ſame till the other, being turned about, do onely touch the Line C M; and where the other Point of the Compaſſes reſteth upon the Line C D, which you will find it to doe at K, draw the Line A K, cutting the Quadrant in P. So ſhall C P, being

being measured upon your Line of Chords, contain 38 degr. 15 min. the quantity of the Angle E, which was required to be found.

Case VII.

The Angle at A 23 degr. 30 min. the Angle at B 122 degr. 36 m. and the Side A B 30 degr. being given, to find the Side B E.

Having let fall the Perpendicular B C, and found the Angle B C E by the former Cases, then

The Analogie or Proportion is,

As the Co-sine of C B E 38 degr. 46 min. is to the Co-sine of A B C 20 degr. 38 min.
So is the Tangent of A B 30 degr. to the Tangent of B E.

THE Quadrants drawn, out of your Line of Chords take 38 degr. 46 min. the Complement of the Angle C B E, and set them from B to S. Also take from your Chords 20 degr. 38 min. the Complement of the Angle A B C, and set them from B to Q, and draw the Lines P Q and R S both parallel to C D. Again, take in your Compasses the distance A R, and set it from C to T. Likewise take the distance from A to P, and setting one foot of that distance in T, with the other describe the Arch Y, and by it draw the Line C M onely to touch the Arch Y. Then take 30 degr. from your Chord, and set them from C to X, drawing the Line A X, and prolonging it to Z. Lastly, from the Point T take the neareſt distance to the Line C M, which distance set from C to V, and draw the Line A V, cutting the Quadrant in Æ. So shall C Æ contain 18 degr. 47 min. of your Line of Chords, which is the quantity of the enquired Side B E.

CASE VIII.

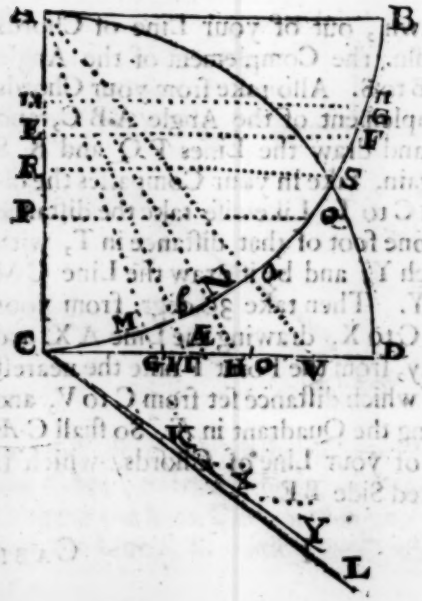
The Side A B 30 degr. the Side B E 18 degr. 47 min. and the Angle at A 23 degr. 30 min. being given, to finde the Angle at B.

Having let fall the Perpendicular B C, and found the Angle A B C 69 degr. 22 min. by the foregoing Cases of Right-angled Spherical Triangles,

The Analogie or Proportion is,

As the Tangent of B E 18 degr. 47 min. is to the Tangent of A B 30 degr.

So is the Co-sine of A B C 20 degr. 38 min. to the Co-sine of C B E.



HAVING drawn your Quadrants, out of your Line of Chords take 18 degr. 47 min. the Side B E, and set them from C to M. Also take 30 d. the quantity of the Side A B, from your Chord, and set them from C to N; and draw the Lines A M and A N, continuing them to G and H. Likewise take 20 degr. 38 min. out of your Line of Chords, and set them from B to F, and draw F E parallel to A B.

This

This done, take in your Compasses the distance C G, and setting one foot in the Point H, with the other describe the Arch K, and draw the Line C L onely to touch it. Then take in your Compasses the distance A E, and setting one foot of the Compasses upon the Line C D, move it along the same gently, till the other, being turned about, do onely touch the Line C L; and where the Compass-point resteth, which you will finde it to doe at O, make a mark, and take the distance O C in your Compasses, and set it from A to P, and draw the Line P Q parallel to A B. So shall B Q, measured upon the Line of Chords, contain 36 degr. 46 min. the Complement of the Angle C B E; or, the distance C Q, measured upon the Chords, will give 53 degr. 14 min. the Angle C B E it selfs which being added to 69 degr. 22 min. the Angle A B C, the Sum will be 122 degr. 36 min. the quantity of the whole Angle A B E required.

CASE IX.

The Angle at A 23 degr. 30 min. the Angle at E 38 degr. 15 m. and the Side B A 30 degr. being given, to find the Side E A.

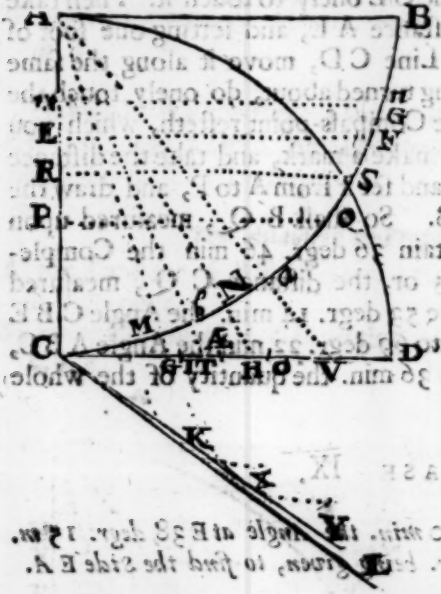
The Perpendicular being let fall, and A C 27 degr. 34 min. a part of the Side A E; then

The Analogie or Proportion is,

As the Tangent of the Angle at E 38 degr. 15 min. is to the Tangent of the Angle at A 23 degr. 30 min.
So is the Sine of A C 27 degr. 34 min. to the Sine of E C.

HAving drawn your Quadrants, take out of your Line of Chords 38 degr. 15 min. the quantity of the Angle at E, and set them from C to *a*. Also take 23 degr. 30 min. the quantity of the Angle at A, and set them from C to *b*, drawing the Lines A *b* and A *a*, and continuing of them to T and V.

This done, take in your Compasses the distance C T, and setting one foot of them in V, with the other describe the Arch X, and draw the Line C Y onely to touch the Arch X. Then take in your Compasses the distance A R, and set it from C to Æ, and from the Point Æ take the least distance to the Line C Y, which distance set from A to *m*, and draw the Line *m n* parallel to A B. So B *n*, being measured upon your Line of Chords, shall contain 14 degr. 57 min. a portion of the Side A E, which being added to the other portion A C 27 degr. 54 min. the Sum of them is 42 degr. 51 min. the whole Side A E, which was required.



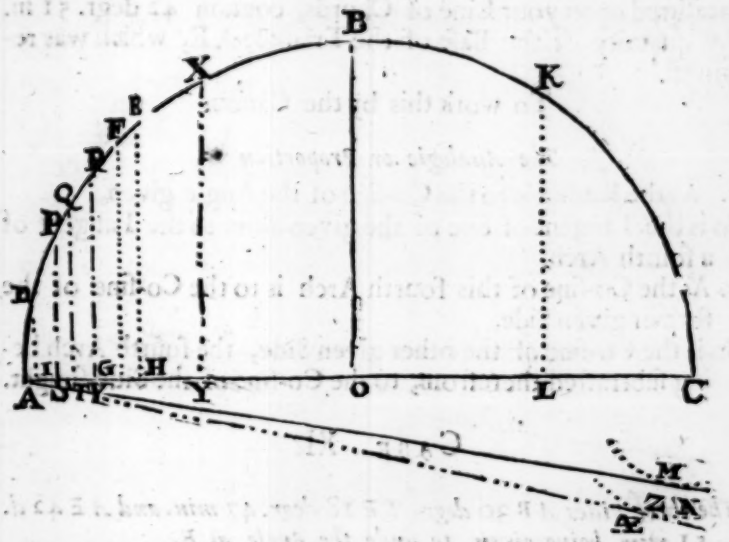
Case X.

The Side A B 30 degr. the Side B E 18 d. 47 m. and the Angle at B 120 degr. contained by them being given, to find the Base.

Take the Sum and the Difference of the two Sides A B, 30 d. and B E, 18 degr. 47 min.

	degr.	m.
Their Sum is	48	47
Their Difference is	11	13

First, draw a right Line A O C, and upon the Centre O describe the Semicircle A B C, drawing the Semidiameter or Perpendicular B O.



Being thus prepared, take out of your Line of Chords 48 d. 47 min. the Sum of the two Sides given, and set them upon the Semicircle from A to E. Also take 11 degr. 30 min. the Difference of the two given Sides, and set them from A to D. Again, take 122 degr. 36 min. the quantity of the given Angle, and set them from A to K; [but A B being 90 degr. set the residue, namely, 32 degr. 26 min. from B to K,] and draw the Lines D I, E H, and K L, all of them perpendicular to the Line A C.

This done, take in your Compasses the distance between I and H, and setting one foot of that extent in C, with the other describe the Arch M; and draw the Line A N so that it

t may onely touch the Arch M. Then ſetting one foot of the Compaſſes in L, with the other take the neareſt diſtance you can from L to the Line A N, and ſet that diſtance from I to G. Laſtly, upon the Point G erect the Perpendicular G F, cutting the Semicircle in F. So ſhall the diſtance A F, being meaſured upon your Line of Chords, contain 42 degr. 51 m. the quantity of the Baſe of the Triangle A E, which was required.

To work this by the Canon.

The Analogie or Proportion is,

1. As the Radius is to the Co-ſine of the Angle given,
So is the Tangent of one of the given Sides to the Tangent of a fourth Arch.
2. As the Co-ſine of this fourth Arch is to the Co-ſine of the former given Side,
So is the Co-ſine of the other given Side, the fourth Arch being ſubtracted therefrom, to the Co-ſine of the Side ſought.

Case XI.

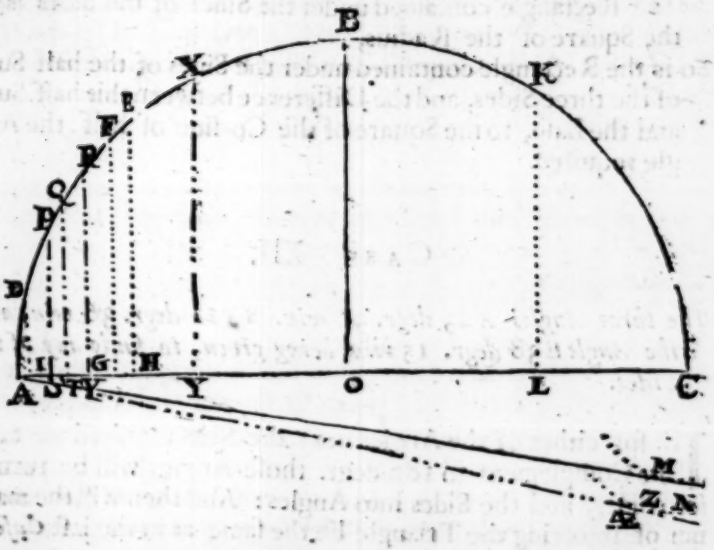
The three Sides A B 30 *degr. B E* 18 *degr.* 47 *min. and A E* 42 *d.* 51 *min. being given, to finde the Angle at E.*

First, find the Sum and the Difference of the Sides B E and A E.

	degr.	m.
Their Sum is	61	38
Their Difference is	24	04

THE Sum and Difference of the two Sides being taken, out of your Line of Chords take 24 degr. 4 min. the Difference of them, and ſet them from A to P. Alſo from your Line of Chords take 61 degr. 38 min. the Sum of the two Sides, and ſet them from A to X. Again, take 30 degr. the

the quantity of the third Side A B, and ſet them from A to Q; and draw the Lines P S, Q T, and X Y, all three perpendicular to A C.



This done, take the diſtance between Y and S, and ſetting one foot of the Compaſſes in C, with the other deſcribe the Arch Z, and draw the Line A Æ ſo that it may onely touch the Arch Z. Then take in your Compaſſes the diſtance between S and T, and ſetting one foot thereof upon the Line A C, move it gently along the ſame, till the other foot, being turned about, do onely touch the Line A Æ; and where the Compaſs-point reſteth, which you will find it to doe at the Point Y, upon this Point Y erect the Perpendicular Y R. So ſhall the diſtance A R, being meaſured upon your Line of Chords, give 38 degr. 15 min. the quantity of the Angle at E, which was required to be found.

To

To work this by the Canon.

The Analogie or Proportion is,

As the Rectangle contained under the Sines of the Sides is to the Square of the Radius,

So is the Rectangle contained under the Sines of the half Sum of the three Sides, and the Difference between this half Sum and the Base, to the Square of the Co-sine of half the Angle required.

Case XII.

The three Angles A 23 degr. 30 min. B 122 degr. 36 min. and the Angle E 38 degr. 15 min. being given, to finde any of the Sides.

IF for either of the Angles next the Side required we take its Complement to 180 degr. those Angles will be turned into Sides, and the Sides into Angles: And then will the manner of resolving the Triangle be the same as in the last *Case.*

Postscript.

Poſtſcript.

BY the Precepts *before in this* Treatiſe *delivered you have the whole* Doctrine *both of* plain Right-lined Triangles, *and alſo of* Spherical *both* Right *and* Oblique-angled, *wrought by a way not uſually practiſed, which if it be carefully performed, you may by a* Line of Chords *of* 3 *or* 4 *inches* Radius *come within a few minutes of* Calculation; *which will be of ſufficient uſe for any Practice either in* Aſtronomicall, Geographicall *or* Nauticall Queſtions, *wrought thereby. And if in the working of any of the* Caſes *there be any difficulty, it is* in the moving of the Compaſſes along one Line, while the other foot, being turned about, doth only touch another Line. *But this is not a real, but a ſeeming, difficulty; for in* 3 *or* 4 *times trial you will finde it very expeditious and eaſie: and although it be not ſo Geometricall as the way by drawing of Parallels, yet it is altogether as exact; more ready and ſpeedy in performance, and avoids the drawing of multiplicity of Lines, which would much cumber the Diagrams to no purpoſe. If the Schemes that are here drawn ſeem to be cumbred with multiplicity of Lines, the reaſon is, becauſe many of them ſerve for* 3, 4, *or* 5 Caſes; *but if a particular Diagram had been drawn for every* Caſe, *then none of them would have conſiſted of above* 5 *Lines beſides the Quadrant, as you will ſee by trying of any* Caſe. *And here note, that every one of theſe* Caſes, *both in* Right-lined *and* Spherical Triangles, *is applicable to ſome one thing or other in the Practice of the* Mathematicks. *As, by* plain Right-lined Triangles *all Propoſitions are reſolved that concern the taking of* Heights, Depths *and* Diſtances, Meaſuring *of* Land, Sailing *by the* plain Sea-Chart, *and alſo by* Mercator's, *and divers other particulars. The Solution of* Spherical Triangles *reſolves Queſtions in* Aſtronomy, Geography, Dialling, Sailing by the Arch of a great Circle, *with many other particulars too tedious here*

to be recited; of ſome of which (eſpecially thoſe that concern the taking of Diſtances *in* Aſtronomy, Geography, Navigation, *&c.) I ſhall in the following Treatiſes exemplifie their Uſe.*

And becauſe the Doctrine *of* Spherical Triangles *is more difficult then that of* Plain Triangles, *I will in this next Treatiſe following ſhew you how (by your Line of Chords onely) you may* draw *or* project *the* Globe *or* Sphere *upon a* Plain; *by which means you may ſee before you how the* Spherical Triangles *lie in the* Sphere *it ſelf; which will be a great help to the underſtanding of the nature of a* Spherical Triangle. *The* Sphere *being thus projected upon a Plain, I will ſhew how (upon that Plain) to meaſure both the* Sides *and* Angles *of a* Spherical Triangle, *and, in ſo doing, reſolve ſome Queſtions in* Aſtronomy; *by which you may diſcern, in that particular, how ſubſervient* Trigonometry *is to* Aſtronomy, *and, as the Proverb is,* judge of *Hercules* by his Foot: *for to inſtance in the Variety of things that the Doctrine of* Triangles *is aſſiſtent to, were an endleſs Work.*

To *project* the

SPHERE

Upon the Plain of the

MERIDIAN

BY

A LINE OF CHORDS.

Whereby the *Sides* and *Angles* of *Spherical Triangles* are naturally laid down *in Plano*, as they are in the *Sphere* it ſelf; By which the *nature* of them is diſcovered, and their *Sides* and *Angles meaſured* with *ſpeed* and *exactneſs*.

The Fourth EXERCISE.

Being to treat of the Projection *of the* Sphere in Plano, *I ſuppoſe the Reader to be acquainted with the* Doctrine of the Sphere *or* Globe, *and with the* Circles *thereof*; *the* nature *of them*, *how they are there* ſituate *one from another in reſpect of diſtance*, *and to what* uſe *each of them is appropriate. But if there be any that hath a deſire to make uſe of this*

Treatiſe, *and is ignorant of the* Sphericks, *let them reade the Books of ſuch as have written of the* Sphere *or* Globe. *In* Latin *there are divers, as* Theodocius, Orontius, Clarius, *&c. In* Engliſh *there is* Record's *Caſtle of Knowledge*, Hill's *School of Skill*, Blundevil's *Exerciſes*, Hews *of the Globes*, Newton, Moxon, *&c. But that this* Treatiſe *may not be accounted deficient in that which is ſo abſolutely neceſſary for the underſtanding and practice of what is herein contained; I will in this place give you a* brief *and* plain *Deſcription of the* Names, Properties, Diſtances, *&c. of ſuch* Circles *of the* Sphere *as in this Book we ſhall have occaſion to* project *for the delineating of* Spherical Triangles in Plano, *and that in a correſpondent Poſition to their ſituation on the* Globe *or* Sphere *it ſelf.*

The Circles *therefore chiefly made uſe of in this* Projection *are theſe:*

1. *The* Meridian
2. *The* Horizon
3. *The* Æquinoctial
4. *The* Ecliptick
5. *The* Prime Verticall, *or* Circle of Eaſt and Weſt
6. *The* Hour-Circles
7. Azimuths, *or* Verticall Circles
8. *The two* Tropicks
9. Parallels *or* Circles *of* Declination
10. Circles *or* Parallels *of* Altitude.

Of theſe Circles *all but the three laſt are* great Circles *of the* Sphere, *which divide it into two equal parts; and the two* Tropicks, *the* Circles of Altitude *and* Declination, *are* ſmaller Circles, *and divide the* Sphere *into two parts unequally.*

Beſides theſe great *and* ſmall Circles, *there are ſeverall* Points *of note upon the* Globe: *as* (1.) *The* Zenith, *which is the Point in the Heavens directly over our heads, in what part of the Earth ſoever we be*; (2.) *The* Nadir, *which is the Point directly under our feet*; (3.) *The* Poles of the World, *about which the Heavens are moved*; (4.) *The Poles of the Ecliptick*; (5.) *The Poles of all other Circles.*

I. *Of*

I. *Of the* MERIDIAN.

THE *Meridian* is a great Circle of the *Sphere*, which paſſeth through both the *Poles* of the *World*, and alſo through the *Zenith* and *Nadir Points*, and croſſeth the *Horizon* in the *North* and *South* Points thereof. Unto this Circle (any Day in the year) when the Sun cometh it is Noon or Mid-day; and when the Moon, Stars or Planets, in the Night come to touch this Circle, they are then ſaid to be upon the *Meridian*, or at the higheſt they will be that Night. This Circle in the Scheme of this Projection is noted by the Letters Z H N O.

II. *Of the* HORIZON.

THE *Horizon* alſo is a great Circle of the *Sphere*, and it is that Circle which divideth the viſible part of the Heavens which we ſee from the not viſible, that is, it divideth the *Sphere* into two *Hemiſpheres*, the lower and the higher. To this Circle when either the Sun, Moon, Stars or Planets, come on the *Eaſt* part, they are then ſaid to *riſe*; and when they have paſſed from the *Eaſterly* Point, by the *Meridian*, and deſcended to the *Weſtern* part of this Circle, they are then ſaid to *ſet*. This Circle is repreſented in the Projection by the right Line H A O.

III. *Of the* ÆQUINOCTIAL.

THE *Æquinoctial* is a great Circle, and in the *Sphere* it is elevated above the *Horizon* (upon the *Meridian* Circle) ſo much as is the Complement of the Latitude of the Place. As at *London*, where the Latitude is 51 degr. 30 min.

there the *Æquinoctial* is elevated 38 degr. 30 min. (which is ſo much as 51 degr. 30 min. wants of 90 degr.) and it cutteth the *Horizon* in the Points of *Eaſt* and *Weſt*. Unto this Circle when the Sun cometh (which is twice every year, namely, about the 10. of *March* and the 12. of *September*) it cauſeth the Daies and Nights to be of equal length all the World over. This Circle is noted in the Scheme with Æ A æ, and cuts the *Horizon* in the Point A, which repreſents both the *Eaſt* and *Weſt* Points thereof.

IV. *Of the* ECLIPTICK.

THIS alſo is a great Circle of the *Sphere*, and (in the *Northern Hemiſphere*, where the *North Pole* is viſible above the *Horizon*, and the *South Pole* not viſible) is elevated above the *Æquinoctial* Circle ſo much as is the Sun's greateſt Declination, which is 23 degr. and about 30 min. and is as much depreſſed below the *Æquinoctial* in the *Southern Hemiſphere*. This Circle is called by ſome *The Way of the Sun*, for that the Sun in his motion never ſwerveth or goeth out thereof, and ſo his *Longitude* or *Place* is counted in this Line. It cutteth the *Horizon* in the *Eaſt* and *Weſt* Point A, as the *Æquinoctial* did. It is repreſented in the *Scheme* by the Line ♋ A ♑, and hath charactered upon it the 12 Signs of the Zodiack; the *ſix Northern Signs*, ♈ ♉ ♊ ♋ ♌ and ♍ being on that half which is above the *Horizon*, and the *ſix Southern Signs* ♎ ♏ ♐ ♑ ♒ and ♓, on the other half, which is below the *Horizon*.

V. *Of the* PRIME VERTICALL.

THE *Prime Verticall*, or Circle of *Eaſt* and *Weſt*, (generally called the *Æquinoctial Colure*, and then (as the *Sphere* is here projected) the *Meridian* repreſenteth the *Solſtitial Colure*)

Colure) is a great Circle passing through the *Zenith* and *Nadir* Points, and also through the *East* and *West* Points of the *Horizon*. Unto this Circle when the Sun, Moon, Stars or Planets, do (in their Motions) arrive, they are then due *East* or *West*. It is in the Projection signified by the right Line Z A N, passing through Z, the *Zenith*, N, the *Nadir*, and A, the *East* and *West* Point of the *Horizon*: and also cutteth the *Æquinoctial* in the Points ♈ and ♎.

VI. *Of the* HOUR-CIRCLES.

THE *Hour-Circles* are great Circles of the *Sphere*, meeting together in the *Poles* of the World, and crossing the *Æquinoctial* at right Angles, dividing it at every 15 degrees; and then every of those Divisions is one Hour of time: but if they pass through other parts of the *Æquinoctial*, dividing it unequally, then do those *Hour-Circles* represent unequal Spaces of time, according to the distance they are from the *Meridian*, or one from another. Of these Circles in the Scheme of the Projection there are four, thus noted P B S, P A S, P C S, and P D S.

VII. *Of the* AZIMUTH CIRCLES.

THE *Azimuth* or *Verticall Circles* are great Circles of the *Sphere*, meeting together in the *Zenith* and *Nadir* Points, as the *Hour-Circles* do in the *Poles* of the World, and divide the *Horizon* at right Angles, either equally, or unequally, as the *Hour-Circles* did the *Æquinoctial*. In the Scheme of the Projection there are four of these *Verticall Circles*, thus noted, Z ☉ N, Z F N, Z A N, and Z G N.

VIII. *Of*

VIII. *Of the* TROPICKS.

THE *Tropicks* are leſſer Circles of the *Sphere*, dividing it unequally, and are drawn parallel to the *Æquinoctial*, at 23 degr. 30 min. diſtance therefrom, equal to the Sun's greateſt Declination on either ſide. That *Tropick* which is on the *North-ſide* is called *The Tropick of Cancer*, to which when the Sun cometh (which is but once in a year, about the 10. of *June*) it maketh the *longeſt Daies* to all the *Northern* Inhabitants of the World, and the *ſhorteſt Nights*. The other *Tropick*, which is on the *South-ſide* of the *Æquinoctial*, is called *The Tropick of Capricorn*, to which when the Sun cometh, which is about the 11. of *December*, it maketh the *ſhorteſt Daies* and the *longeſt Nights* to all *Northern* Inhabitants, and the contrary to all the *Southern* Inhabitants of the World. In the Projection the *Tropick of Cancer* is ſignfied by ♋ I ♋, and the *Tropick of Capricorn* by ♑ K ♑.

IX. *Of the CIRCLES or PARALLELS of DECLINATION.*

THESE alſo are ſmaller Circles of the *Sphere*, and are drawn parallel to the *Æquinoctial*, towards both the *Tropicks*, and up to them. Thoſe that are on the *North-ſide* of the *Æquinoctial* are called *Parallels of North Declination*, and thoſe that are on the *South-ſide* of the *Æquinoctial* are called *Parallels of South Declination*. Of theſe *Parallels* there are in the Scheme of the Projection two, one towards the *Tropick of Cancer*, the other towards the *Tropick of Capricorn*, and either of them 20 degrees diſtant from the *Æquinoctial*. The *Northern Parallel of Declination* is noted with ♊ ☉ ♌, and the *Southern* with ♒ E ♐.

X. *Of*

X. *Of the* CIRCLES *or* PARALLELS *of* ALTITUDE.

THE Circles of *Altitude* are likewiſe ſmall Circles of the *Sphere*, and are drawn parallel to the *Horizon*, as the Circles of *Declination* were to the *Æquinoctial*. Theſe *Parallels* are drawn from the *Horizon* towards the *Zenith* Point, and upon occaſion, in many Caſes, quite up unto it. By theſe *Parallels* are meaſured the *Altitude* or *Height* of the Sun, Moon and Stars. In the Scheme there is onely one of them, and that is expreſſed by the Letters M E L.

Thus have I given you a brief and plain Deſcription *of the* Circles, *both* great *and* ſmall, *which we ſhall have occaſion to uſe in this following* Treatiſe. *And here note, that every* Circle *of the* Sphere (*both great and ſmall*) *hath his proper* Poles, *which* Poles (*of all the* great Circles) *are* 90 *Degrees, or a* Quadrant *of a* Circle, *diſtant from the* Circle *it ſelf. The* Poles *of the* Circles *in this* Projection *are as followeth.*

Z and N, P and S, O and H, Q and R, Æ and *æ* } Are the Poles of {
- H A O, the Horizon.
- Æ A *æ*, the Æquinoctial.
- Z A N, the Prime Verticall.
- the Ecliptick.
- P A S, the Axis of the World.

The Poles *of theſe five* Circles *are all in the* Meridian, *and ſo there needeth no farther Precept for the finding of them*; *and the* Pole *of the* Meridian *is the Centre thereof.*

But for the three Azimuth Circles, *they fall in ſeveral Points of the* Horizon; *and the three* Hour-Circles *in certain Points in the* Æquinoctial. *How to finde which Points ſhall be ſhewed afterwards in due place.*

A T G ⊙ X Y V } Is the Pole of { the Meridian, Z H N O. / the Azimuth Circle, { Z F N. Z ⊙ N. Z G N. / the Hour-Circle { P B S. P D S. P C S.

The Poles of the World P *and* S *are also the* Poles *of the* Tropicks *and of all the* Parallels of Declination. *And*

The Zenith *and* Nadir, Z *and* N, *are the* Poles *of all the* Parallels *of* Altitude.

Having sufficiently acquainted the Reader with the several Circles, Lines, Points *and* Poles, *belonging to every* Circle, *I will now proceed to my intended purpose; namely, to project (or lay down* in Plano*) all these* Circles, Lines, Points *and* Poles, *in their true Positions.*

How to project the Sphere *upon the Plain of the* Meridian.

FIrst, take 60 degr. of your Line of Chords, and with that distance upon the Point A (as a Centre) describe the Circle Z H N O, representing the *Meridian*, (within which Circle all the rest are to be projected) and cross it with the two Diameters H A O the *Horizon*, and Z A N the *Prime Verticall.*

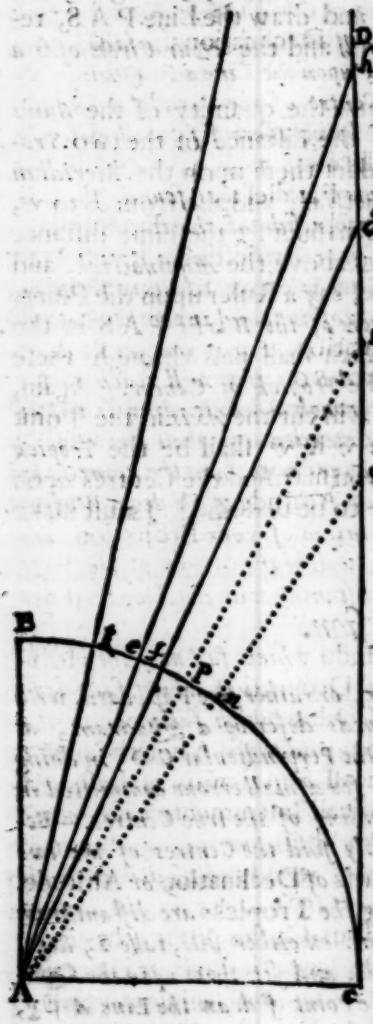
Secondly, (because the *Latitude* of the place for which you draw your *Projection*, viz. *London*, is 51 degr. 30 min.) take 51 degr. 30 min. from your Line of Chords, and set them upon the *Meridian* from Z to Æ, and from N to *æ*, and draw the

the Line Æ A ♈ for the *Æquinoctial.* Alſo ſet 51 degr. 30 m. from O to P, and from H to S, and draw the Line P A S, repreſenting the *Axis of the World* and the *Hour-Circle of* 6 *a clock.*

Thirdly, take 23 degr. 30 min. the quantity of the *Sun's greateſt Declination*, and alſo of the diſtance of the two *Tropicks* from the *Æquinoctial*, and ſet them upon the *Meridian* from Æ to ♋, above the *Æquinoctial*, and alſo from Æ to ♑, below the *Æquinoctial.* In like manner ſet the ſame diſtance of 23 degr. 30 min. from ♈ to ♋ above the *Æquinoctial*, and from ♈ to ♑ below it. This done, lay a Ruler upon the Points Æ and ♋, and it will cut the *Axis of the World* P A S in the Point I. So a Circle drawn which ſhall paſs through theſe three Points, ♋ I ♋, ſhall be the *Tropick of Cancer.* Again, lay a Ruler to Æ and ♑, and it will cut the *Axis* in the Point K. So a Circle drawn through ♑ K ♑ ſhall be the *Tropick of Capricorn.* But to ſhew how you may find the Centres upon which theſe Tropical Circles are to be deſcribed, I muſt make this

Diverſion.

Upon a long piece of ſtiff Paper, or rather fine Paſtboard, with 60 *Degrees of your Line of Chords deſcribe a Quadrant, as A B C, and upon the Point C erect the Perpendicular C D; in doing whereof you muſt be very carefull, for a ſmall errour committed in that will produce a great one in finding of the true Centres.—Being thus prepared, you may readily find the Centres of the two* Tropicks, *and of any other* Parallels *of* Declination *or* Altitude. *But* (1.) *for the* Tropicks: *Being the* Tropicks *are diſtant from the* Æquinoctial 23 *degr.* 30 *min. on either ſide, take* 23 *degr.* 30 *min. out of your Line of Chords, and ſet them upon the* Quadrant *from B to f, and through the Point f draw the Line A f g, cutting the Perpendicular C D in the Point g. This done, take*



in your Compasses the distance C g, and setting one foot in the Point I in your Projection, where the other Point falleth upon the Line I P, it being sufficiently extended, will be the Centre of the Tropick of Cancer; *and a Circle described with this distance of the Compasses, C g, will pass exactly through the Points ☋ I ☋, and so describe your* Tropick of Cancer. *The like you must doe for the* Tropick of Capricorn, *by taking the distance C g in your Compasses, and setting one foot in the Point K of your Projection, and where the other reacheth upon the Line K S, being extended, there is the Centre of the* Tropick of Capricorn. *But* (2.) *to finde the Centres of any of the other* Parallels of Declination, *as of the two in the Projection, namely,* ♊ *a* ♌ *and* ♏ *e* ♐, *either of which are* 20 *d. distant from the* Æquinoctial; *the Centres of them are also found in the same manner as the Centres of the* Tropicks *were. For take* 20 *degr. out of your*

Line

Line of Chords, and ſet them upon the Quadrant *from B to e, drawing the Line A e h, the diſtance C h ſhall be the* Semidiameter *of the Parallel of* 20 *degr. of* Declination; *which, being ſet upon your* Projection *from a, upon the Line a P, (being extended) will there give you the Centre of the* Parallel; *and the Compaſſes at the diſtance of C h, being there ſet, will deſcribe the* Parallel *of* 20 *degr. of* Declination π *a* ☊. *And in the ſame manner that the Centre of this* Parallel *was found, ſo may you find the Centre of that on the other ſide of the* Æquinoctial, ♎ *c* ♈. *But* (3.) *you have in your* Projection *a* Parallel *or* Circle *of* Altitude, *namely, M E L, which is* 12 *degr. from or above the* Horizon, *the Centre whereof is found in the ſame manner as were the* Parallels *of* Declination. *For if you ſet* 12 *degr. of your Chord upon the* Quadrant *A B C, they will reach to k: draw a Line therefore from A through k, extending it till it meet with the Line C D being alſo extended; ſo ſhall the diſtance between this Point, where the two Lines meet, and the Point C, be the* Semidiameter *of the* Parallel *of* Altitude *of* 12 *degr. Wherefore that diſtance ſet from the Point m of your* Projection, *upon the Line m Z, being extended, will be the Centre of the* Parallel, *and the Compaſſes, at that diſtance, will deſcribe the* Parallel *of* Altitude *M m L.*

But for thoſe Parallels *of* Altitude *which fall near the* Horizon, *thoſe* Circles *or* Parallels *of* Declination *which fall near to the* Æquinoctial, *thoſe* Hour-Circles *which fall near to the* Axis *of the* World *or* Hour *of* Six, *and thoſe* Azimuth Circles *which are near to the* Prime Verticall *or* Azimuth *of* Eaſt *and* Weſt; *thoſe that make* Mathematicall Inſtruments *have an Inſtrument called a* Bow, *which, by the help of one or more Screws, (according to the length of the* Bow) *may be extended to touch any three Points which lie near in a ſtraight Line; by the edge of which* Bow *you may draw your* Hour-Circles, Azimuths, Parallels *of* Declination *and* Altitude, *as eaſily as you may draw a right Line by the edge of a Ruler.* ——*But to return again to our* Projection.

Fourthly, draw a right Line ♋ A ♑ between the two *Tropicks*, touching the *Tropick of Cancer* above the *Horizon* at ♋, and the *Tropick of Capricorn* below the *Horizon* at the Point ♑.—This Circle hath upon it the Characters of the 12 Signs of the *Zodiac*, which are to be put on in this manner.—Take 23 d. 30 min. out of your Line of Chords, and ſet them from P to Q, and from ♋ to R: which Points, Q and R, are the two *Poles* of the *Ecliptick*. Then take 60 degr. from your Line of Chords, and ſet them from Q to 1, and from Q to 3. Alſo ſet the ſame diſtance from ♋ to 2, and from ♑ to 4.—This done, lay a Ruler to the Pole R and the figure 1, it will cut the *Ecliptick* in the Point ♈ and ♎: the Ruler laid to R and 2 will cut it in the Point ♉ and ♍; and laid to R and 4, in ♏ and ♐; and laid to R and 3, in ♊ and ♑. So have you the true Points for the Sun's entrance into every Sign. And if you would have every tenth degree of each Sign, divide every of the Spaces ♋ 1, 1 2, 2 Q, Q 4, 4 3, and 3 ♑, into three equall parts; ſo will each part contain 10 d. and a Ruler laid to each of them and the Point R ſhall give you the Points upon the *Ecliptick* anſwering to the 10. degr. of every Sign. And in the ſame manner may you (if your Projection be large) put on every Degree.

Fifthly, for the putting on of the *Hour-Circles*; conſider how far the Circle you are to put on is diſtant from the *Meridian*, and ſet ſo many degrees upon the *Meridian* from the *Æquinoctial*: a Ruler laid from Z to thoſe degrees will croſs the *Æquinoctial*, and through that Point in the *Æquinoctial* where the Ruler ſo croſſeth, the *Hour-Circle* will paſs.—Example: The *Hour-Circle* P B S, in this Projection, is diſtant from the *Meridian* 62 d. 46 m. wherefore take 62 d. 46 m. from your Chords, and ſet them from *a* to *b*; then laying a Ruler from Z to *b*, it will cut the *Æquinoctial* in B, through which Point the *Hour-Circle* of 62 d. 46 m. muſt paſs.—To find the Centre of this *Hour-Circle*, (and ſo of any other) repair to the former Scheme for finding of the Centres of the *Parallels* of *Altitude* and *Declination*;

clination; and (becauſe this *Hour-Circle* is diſtant from the *Meridian* 62 degr. 46 min.) take 62 degr. 46 min. from your Line of Chords, and ſet them upon the Quadrant A B C, from C to *l*, and draw the Line A *l m*. So ſhall the Line A *l m* be the Semidiameter of the *Hour-Circle* P B S; which being taken in your Compaſſes, and ſet upon your Projection from B, upon the Line B Æ, (being extended,) ſhall there give you the Centre of that *Hour-Circle*. And in the ſame manner may the Centres of all the reſt be found.

Sixthly, the *Azimuth Circles* are to be drawn upon the Projection, and the Centres of them found in all reſpects as the *Hour-Circles* were.——So the *Azimuth Circle* Z ⊙ N, being 56 degr. 40 min. from the *Meridian*, take 56 degr. 41 min. out of your Line of Chords, and ſet them upon the *Meridian* of your Projection from O to *d*; then laying a Ruler unto Z and *d*, it will cut the *Horizon* in the Point ⊙; through which the *Azimuth* of 56 degr. 41 min. Z ⊙ N, muſt paſs.——Then, to find its Centre, repair to the former Scheme for finding of Centres, and upon the Quadrant A B C ſet 56 degr. 41 min. of your Chords, from C to *n*, and draw the Line A *n o*: ſo ſhall the Line A *n o* be the Semidiameter of the *Azimuth Circle* Z ⊙ N; which being taken in your Compaſſes, and ſet upon your Projection from ⊙, upon the Line ⊙ H, (being extended,) ſhall there give you the Centre of the *Azimuth Circle* Z ⊙ N. And in this manner may the Centres of any other *Azimuth Circle* be found.

And here note (1.) *That the Centres of all* Azimuth Circles *fall in the* Horizon, H A D, *being extended where need is. The Centres of all the* Hour-Circles *fall in the* Æquinoctial *Line* E Æ Q, *being extended. The Centres of the* Tropicks *and* Parallels *of* Declination *fall in the* Axis of the World P A S, *extended. And the Centres of the Circles of* Altitude *fall in the* Prime Verticall *Circle* Z A N.

Note (II.) *That if the middle Point of any* Hour-Circle *do not fall juſt in the* Æquinoctial, *or any* Azimuth Circle *juſt in the*

the Horizon, *but on either side of them; then you may find the Centres by the* Geometricall Propositions *at the beginning of this Book; though there be other waies to find the Centres upon the* Projection *it self, which I omit, for that I would not cumber the Scheme with unnecessary Lines.*

Seventhly, Every Circle in the Projection hath its proper *Pole*, as was before intimated. Now for the finding of them, you are to note, that the *Pole* of every *great Circle* is 90 degr. or a *Quadrant* of a Circle, distant from the Circle it self, upon that Line which cutteth the Circle at right Angles. Thus the *Poles* of all the *Hour-Circles* are upon the *Æquinoctial*, and the *Poles* of all the *Azimuths* upon the *Horizon*.——Now if you would find the *Pole* of the *Hour-Circle* P D S, lay a Ruler upon P and D, and it will cut the *Meridian* Circle in *e*: then take 90 degr. of your Line of Chords, and set them from *e* to *f*, a Ruler laid from P to *f* will cut the *Æquinoctial* in Y: so is Y the *Pole* of the *Hour-Circle* P D S.

Lastly, The finding of the *Poles* of the *Azimuth Circles* is the same with the *Hour-Circles*. So if you would find the *Pole* of the *Azimuth Circle* Z G N, lay a Ruler upon Z and G, it will cut the *Meridian* Circle in *g*; then set 90 degr. of your Chord from *g* to *d*, so a Ruler laid from Z to *d* will cut the *Horizon* H A O in the Point ⊙, which Point ⊙ is the *Pole* of the *Azimuth Circle* Z G N. And thus have you found the *Poles* of one of the *Hour* and one of the *Azimuth* Circles. And by the same manner of Work you may find the *Poles* of all the rest. As

The Pole of the	Hour-Circle	P D S, P C S, P A S, P B S	will be found at	Y, V, Æ or *e*, X
	Azimuth Circle	Z G N, Z A N, Z F N, Z ⊙ N	will be found at	⊙, H or O, F, G

The

The Poles of the { Horizon — H A O } are { Z and N, the Zenith and Nadir. / Æquinoctial — Æ A ♎ } are { P and S, the Poles of the World. / Ecliptick — ♎ A ♈ } are { Q and R.

Thus have I given you at large a plain and eaſie method how to project *the* Sphere *upon the Plain of the* Meridian Circle, *by help of the Line of Chords onely: Upon which* Projection, *by the interſection or croſſing of the ſeverall* Circles *thereof, are conſtituted divers* Spherical Triangles; *ſome Right-angled, and others Oblique-angled. By the reſolving of which* Triangles *variety of* Queſtions *appertaining to* Aſtronomie, Geographie *and* Navigation, *may (with ſpeed and exactneſs) be reſolved. But before I come to ſhew the manner of working* particular Queſtions *of any kind, it will be expedient that I ſhew you,* (1.) *how to meaſure or find the quantity of the* Sides *and* Angles *of a* Spherical Triangle, *as they are here projected;* *and* (2.) *how to* project *or* lay down *an* Angle *or* Side *of any quantity that ſhall be required.*

I. *A* Spherical Triangle *being projected, how to find the quantity of any* Angle *thereof.*

LAY a Ruler to the angular Point, and the extremity of the Sides containing the Angle, they being continued to Quadrants; and note where the Ruler cuts the *Meridian* or outward Circle; at both which places make marks upon the *Meridian:* the diſtance between thoſe two marks, being meaſured upon your Line of Chords, ſhall give you the quantity of the Angle required.

Example I.

IN the Triangle P ⊙ O, in the Projection, let it be required to find the quantity of the Angle ⊙ P O.——First, lay a Ruler upon the angular Point P, and to the extreme ends of the Sides P ⊙ and P O, they being extended to Quadrants, which is, to that Circle which measures that Angle: (as the *Æquinoctial* measures all the Angles at P, the *Pole of the World*; the *Horizon* all the Angles at Z, the *Zenith*, &c.) So the Ruler laid from P to *æ*, will cut the *Meridian* in *æ*; and being laid from P to B, it will cut the *Meridian* in the Point *b*. The distance *b æ*, being taken in your Compasses and measured upon your Line of Chords, will be found to contain 62 degr. 46 min. which is the quantity of the Angle ⊙ P O.——But if upon the Point P you were to project an Angle to contain 62 degr. 46 min. then take 90 degr. of your Chords, and set them from P to *æ*, and through the Centre A draw the Line Æ A *æ*; then take 62 degr. 46 m. out of your Line of Chords, and set them from *æ* to *b*; and laying a Ruler from P to *b*, it will cut Æ A *æ* in the Point B: the Circle P B S being drawn, the Angle at P will contain 62 degr. 46 min.

Example II.

LET it be required to find the quantity of the Angle Z E P.——Lay a Ruler to ⊙, the *Pole* of the Circle Z E N, and the Point E, it will cut the *Meridian* Circle in M; from M set 90 degr. to ϰ; a Ruler laid from ⊙ to ϰ will cut the Circle Z E N (it being extended beyond the *Zenith* Z) at the Point ♪.

Again, Lay a Ruler upon Y, the Pole of the Circle P E S, and it will cut the *Meridian* Circle in *v*; set 90 degr. from *v*

to

to *x* upon the *Meridian*; a Ruler laid from Y to *x* will cut the Circle P E S in *y*.

This done, lay a Ruler from E to ♑, and it will cut the *Meridian* in θ; alſo lay the Ruler from E to *y*, it will cut the *Meridian in* λ: the diſtance θλ, being taken in your Compaſſes and applied to your Line of Chords, will be found to contain 21 degr. 45 min. And ſuch is the quantity of the Angle Z E P.

Theſe two ſorts of Angles *are the moſt troubleſome to find their quantities, and therefore I have inſtanced in them. There are other* Angles *in the* Projection *which render their meaſures to the eye, without farther Inſtructions for finding their quantities.*

II. *A* Spherical Triangle *being projected, to find the quantity of any* Side *thereof.*

A Ruler laid upon the Pole of the Circle which is to be meaſured, and to the extreme ends of the Side of the Triangle; note where the Ruler, ſo laid, cuts the *Meridian* at both ends of the Side: that diſtance, taken in your Compaſſes and meaſured upon the Line of Chords, will give you the quantity of the Side of the Triangle.

Example I.

LET it be required to find the Side E Z of the Triangle Z E P.——Lay a Ruler to ⊙ (the Pole of the Circle Z E N) and the angular Point E, it will cut the *Meridian* in M; and a Ruler laid to Z will cut the *Meridian* in Z. So the diſtance M Z, taken in the Compaſſes and meaſured upon the Line of Chords, will be found to contain 78 degr. And ſuch is the quantity of the Side Z E.

Example II.

LET it be required to find the Side ⊙B of the Spherical Triangle A⊙B.—Lay a Ruler upon X, the Pole of the Circle PBS, and the Point B, it will cut the Meridian Circle in *æ*.—Alſo lay a Ruler from X to ⊙, it will cut the Meridian in the Point ♌. The diſtance between *æ* and ♌, being taken and meaſured on the Line of Chords, will contain 20 d. And ſuch is the quantity of the Side ⊙B.

I could inſtance in divers other Examples *concerning the Meaſuring of the* Sides *and* Angles *of* Triangles *upon the* Projection; *but I here omit them, becauſe in the reſolving of the following* Propoſitions *they will come in practice, and the Manner of the performance is there plainly expreſſed: onely I deemed it convenient here to give ſome taſte thereof, as a Preparative to that which followeth.———But before I come to ſhew the Manner of reſolving of particular Queſtions in* Aſtronomie, Geography, *&c. I will declare the Variety of* Spherical Problems *that will naturally ariſe out of every* Spherical Triangle, *being projected.*

THE

THE VARIETY OF *SPHERICALL* PROBLEMS

Naturally arising out of every *Spherical Triangle*, both *Right* and *Oblique-angled*, and that are resolvable thereby, described as they are perspicuous to the Eye in the *Projection*.

The Fifth EXERCISE.

IN *the foregoing Part of this Book you have the* Doctrine *of* Plain *and* Spherical Triangles Geometrically *performed. And in the Solution of* Right-angled Spherical Triangles *there were* 16 Cases; *and in* Oblique-angled *there were* 12 Cases: *but the* 16 Cases *of* Right-angled Triangles *will by this* projective *way be reduced to* 5 Cases, *and the* 12 *of* Oblique-angled *will be reduced to* 6; *so that in both there will be but* 11 Cases, *whereas before there were* 28. *That this may appear plain to the Reader, I will make use of* two

Triangles *in the* Projection; *one whereof ſhall be* Right-angled, *as the* Triangle P O ⊙, Right-angled *at* O; *and the other ſhall be the* Oblique-angled Triangle Z E P.——*The* Right-angled Triangle *is conſtituted by the Interſection of three* great Circles *of the* Sphere; *namely, of* P O, *an Arch of the* Meridian, ⊙ O, *an Arch of the* Horizon, *and* P ⊙, *the Arch of an* Hour-Circle.——*The* Oblique-angled Triangle, Z E P, *is conſtituted alſo of three Arches of* great Circles *of the* Sphere, (*as all* Spherical Triangles *whatſoever are*;) *namely, of* Z P, *an Arch of the* Meridian, P E, *an Arch of an* Hour-Circle, *and* Z E, *an Arch of an* Azimuth Circle.

In the Right-angled Spherical Triangle, P ⊙ O,

The Side
- P O *is the* Latitude of the Place.
- ⊙ O *is the* Sun's Amplitude from the North.
- P ⊙ *is the* Sun's diſtance from the Pole, *or the* Complement of his Declination.

The Angle
- ⊙ P O *is the* Hour from Midnight, *or from the* North part of the Meridian.
- P ⊙ O *is the Angle of the* Sun's Poſition *at the time of the Queſtion.*
- P O ⊙ *is the* Right Angle.

The Parts of the Triangle *being declared, and of what* Circles *of the* Sphere *the* Sides *do conſiſt*; *I will now come to the* Caſes, *which* (*as I ſaid before*) *are* 5 *in every* Right-angled Triangle. *So that any two parts of the* Triangle (*beſides the* Right Angle) *being given, I will ſhew in every of the* 5 Caſes *what parts may be found.*

I. *In*

I. *In a Right-angled Spherical Triangle.*

Case I.

The Base and Perpendicular being given, to finde the other parts of the Triangle.

IN the Triangle P ⊙ O here is given P O, the Latitude, and ⊙ O, the Amplitude from the North part of the *Meridian*; by which you may find

3. {
1. P ⊙, *the Complement of the Sun's Declination.*
2. ⊙ P O, *the Hour from Midnight.*
3. P ⊙ O, *the Angle of the Sun's Position.*

Case II.

The Hypotenuse and Perpendicular being given, to find the other parts.

IN the Triangle P ⊙ O here is given P O, the Latitude, and P ⊙, the Complement of the Sun's Declination; by which may be found

6. {
1. ⊙ O, *the Amplitude from the North.*
2. O P ⊙, *the Hour from Midnight.*
3. P ⊙ O, *the Angle of the Sun's Position.*

And if in stead of the Perpendicular there had been given the Base ⊙ O, you might then find

4. P O, *the Latitude.*
5. ⊙ P O, *the Hour from Midnight.*
6. P ⊙ O, *the Angle of the Sun's Position.*

Case

Case III.

The Hypotenuse and an Angle being given, to find the other Parts.

HERE is given in the Triangle P ⊙ O, the Complement of the Sun's Declination, P ☉ and ⊙ P O, the Hour from Midnight; by which may be found

1. ⊙ O, *the Amplitude from the North.*
2. P O, *the Latitude.*
3. C ⊙ O, *the Angle of the Sun's Position.*

6. And if in lieu of the Angle at P, the Angle at ⊙ P had been taken, then you might have found

4. ⊙ O, *the Amplitude from the North.*
5. P O, *the Latitude.*
6. ⊙ P A, *the Hour from Midnight.*

Case IV.

The Perpendicular, or Base, and either of the Angles given, to find the other Parts.

IN the Triangle P ⊙ O let there be given the Amplitude, ⊙ O, and the Angle of the Sun's Position, P ⊙ O; by which you may find

1. P O, *the Latitude.*
2. ⊙ P O, *the Hour from Midnight.*
3. P ⊙, *the distance of the Sun from the Pole.*

But if the Side given had been ⊙ O, and the Angle given ⊙ P O, then you might have found

4. P ⊙, *the Complement of the Sun's Declination.*
5. P O, *the Latitude.*
6. P ⊙ O, *the Angle of the Sun's Position.*

12. But again, if ⊙ O and ⊙ P O had been given, then might be found

7. ⊙ P, *the Complement of the Sun's Declination.*
8. ⊙ O, *the Amplitude from the North.*

9. P ⊙ O,

9. P ⊙ O, *the Angle of the Sun's Position.*
And again, if P O and O ⊙ P had been given, we might then have also found
10. ⊙ P, *the distance of the Sun from the Pole.*
11. ⊙ O, *the Amplitude from the North.*
12. ⊙ P O, *the Hour from Midnight.*

Case V.

The Angles being given, to find the other Parts.

If the two Angles P ⊙ O and O P ⊙ be given, there may be found

3. 1. P O, *the Latitude.*
2. ⊙ O, *the Amplitude from the North.*
3. P ⊙, *the Complement of the Sun's Declination.*

Thus you see, that in this one Right-angled Spherical Triangle, *by the several Parts given in these* five Cases, *there are* 30 Spherical Problems *resolved; and so many are resolvable in every* Right-angled Triangle.

II. *In an Oblique Spherical Triangle.*

In the Oblique-angled Spherical Triangle, Z P E,

The Side
- Z P is the *Complement of the Latitude.*
- P E is the *Complement of the Sun's Declination*, or his distance from the *North Pole.*
- Z E is the *Complement of the Sun's Altitude.*

The Angle
- E Z P is the *Sun's Azimuth from the North part of the Meridian.*
- Z P E is the *Hour from Noon.*
- Z E P is the *Angle of the Sun's Position.*

Case I.

The three Sides being given, to find an Angle.

IN the Triangle Z E P, if there be given the Side E Z, the Complement of the Sun's Altitude, Z P, the Complement of the Latitude, and E P, the Sun's diſtance from the Pole, or the Complement of his Declination, you may find

3. 1. E Z P, *the Sun's Azimuth from the North.*
   2. Z E P, *the Angle of the Sun's Poſition.*
   3. Z P E, *the Hour from Noon.*

Case II.

Two Sides and the Angle comprehended by them being given, to find the other Parts of the Triangle.

IF in the Triangle Z E P there be given the Side E Z and Z P, and the Angle between them E Z P, you may find

9. 1. Z E P, *the Angle of the Sun's Poſition.*
   2. Z P E, *the Hour from the South, or Noon.*
   3. E P, *the Sun's diſtance from the Pole.*

   But if the Sides Z P and P E, and the Angle Z P E between them, had been given, then might have been found

   4. P E Z, *the Angle of the Sun's Poſition.*
   5. E Z, *the Complement of the Sun's Altitude.*
   6. E Z P, *the Azimuth of the Sun from the North.*

   And if the Sides Z E and P E, with the Angle Z E P contained by them, had been given, there might be found

   7. E Z P, *the Sun's Azimuth from the North.*
   8. Z P, *the Complement of the Latitude.*
   9. Z P E, *the Hour from Noon.*

Case

Case III.

Two Angles, and a Side contained by them, being given, to find the other Parts.

IF in the Triangle Z E P the Angle E Z P and the Angle Z P E, with the Side contained between them, Z P, be given, we may find

1. Z E, *the Complement of the Sun's Altitude.*
2. Z E P, *the Angle of the Sun's Position.*
3. E P, *the Complement of the Sun's Declination.*

But if the Side E P, and the Angles Z E P & Z P O, had been given, then might be found

4. E Z, *the Complement of the Sun's Altitude.*
5. E Z P, *the Azimuth from the North.*
6. Z P, *the Complement of the Latitude.*

And if the Side Z E, and the Angles P Z E and P E Z, had been given, then you might find

7. Z P, *the Complement of the Latitude.*
8. Z P E, *the Hour from Noon.*
9. P E, *the Sun's distance from the Pole.*

Case IV.

Two Sides, with an Angle opposite to one of them, being given, to find the other Parts.

IF there be given the Side Z P, the Side E P, and the Angle Z E P, there may be found

1. E Z, *the Complement of the Sun's Altitude.*
2. E Z P, *the Sun's Azimuth from the North.*
3. Z P E, *the Hour from the South.*

But if the Side Z P, and the Side E P, with the Angle E Z P, had been given, then might be found

4. E Z, *the Complement of the Sun's Altitude.*

5. Z E P, *the Angle of the Sun's Position.*
6. Z P E, *the Hour from Noon.*

And if there had been given E P, E Z, and E Z P, then might be found

7. Z P, *the Complement of the Latitude.*
8. Z P E, *the Hour from Noon.*
9. Z E P, *the Angle of the Sun's Position.*

In like manner, if the Sides P E and E Z, with the Angle Z P E, had been given, then might be found

18.

10. Z P, *the Complement of the Latitude.*
11. E Z P, *the Azimuth from the North.*
12. Z E P, *the Angle of the Sun's Position.*

Again, if the Sides E Z and Z P, with the Angle Z P E, had been given, then would be found

13. P E, *the Sun's distance from the Pole.*
14. Z E P, *the Angle of the Sun's Position.*
15. E Z P, *the Sun's Azimuth from the North.*

Lastly, if the Sides E Z and Z P, with the Angle Z E P, had been given, then you might find

16. P E, *the Complement of the Sun's Declination.*
17. Z P E, *the Hour from Noon.*
18. E Z P, *the Azimuth from the North.*

Case V.

Two Angles, and a Side opposite to one of them, being given, to find the other Parts of the Triangle.

In the Triangle Z E P, if there be given the Angles E Z P and Z P E, with the Side P E, there may be found

1. Z P, *the Complement of the Latitude.*
2. Z E, *the Complement of the Sun's Altitude.*
3. Z E P, *the Angle of the Sun's Position.*

But if there were given E Z P and Z P E, with the Side Z E, then might be found

4. Z P,

4. Z P, *the Complement of the Latitude.*
5. P E, *the Sun's diſtance from the Pole.*
6. Z E P, *the Angle of the Sun's Poſition.*

And if the Angles Z P E and Z E P, with the Side E P, were given, then might be found

7. Z P, *the Complement of the Latitude.*
8. P E, *the Hour from Noon.*
9. E Z P, *the Sun's Azimuth from the North.*

18. Again, if there were given the Angles Z P E and Z E P, with the Side Z P, you might then find

10. Z E, *the Complement of the Sun's Altitude.*
11. P E, *the Complement of the Sun's Declination.*
12. E Z P, *the Sun's Azimuth from the North.*

Alſo if there were given the Angles Z E P and E Z P, with the Side Z P, you might find

13. Z E, *the Complement of the Sun's Altitude.*
14. P E, *the Complement of the Sun's Declination.*
15. Z P E, *the Hour from Noon.*

And laſtly, if there were given the Angles Z E P and E Z P, with the Side P E, then might be found

16. Z E, *the Complement of the Sun's Altitude.*
17. Z P, *the Complement of the Latitude.*
18. Z P E, *the Hour from Noon.*

Case VI.

The three Angles being given, to find the other Parts.

In the Triangle Z E P, if the three Angles E Z P, Z P E, and P E Z, be given, there may be found

3. 1. Z P, *the Complement of the Latitude.*
   2. P E, *the Sun's diſtance from the Pole.*
   3. Z E, *the Complement of the Sun's Altitude.*

Thus have you in theſe ſix Caſes all the Varieties that will ariſe out of an Oblique-angled Spherical Triangle, in the Converſion of

which Cases you may observe 60 Questions of the Sphere to be resolved; and so many are resolvable in every Oblique-angled Spherical Triangle, and 30 in every Right-angled: So that in these two Triangles 90 Questions are resolved. For,

In a Right-angled Spherical Triangle,

By the	First	Case are resolved	3	Spherical Questions.
	Second		6	
	Third		6	
	Fourth		12	
	Fifth		3	

In all 30.

In an Oblique-angled Spherical Triangle,

By the	First	Case are resolved	3	Spherical Problems.
	Second		9	
	Third		9	
	Fourth		18	
	Fifth		18	
	Sixth		3	

In all 60.

PRO-

PROPOSITIONS
ASTRONOMICALL,
Uſefull in the Practice of
NAVIGATION:

Performed by the reſolving of ſeverall *Spherical Triangles* upon the *Projection.*

The Sixth EXERCISE.

OF *the vaſt number of* Spherical Queſtions *that are reſolvable by the ſeverall* Spherical Triangles *that are* (*by the Interſections of the* great Circles *of* the Sphere) *conſtituted upon the* Projection, *ſeeing there may be* 30 *in every* Right-angled, *and* 60 *in every* Oblique-angled Spherical Triangle *reſolved*; *I have made choice onely of ſuch as I conceived moſt neceſſary for* Sea-mens *uſe. Of which, ſome of them will be aſſiſtent to them in finding the* Latitude; *others for finding the* Sun's Altitude *upon ſome certain* Points *of*

of the Compaſs, *and at ſome particular* Hours: *others will help them to find the* Variation of their Compaſs; *and ſome of them will help them to the* Hour of the Day and Night.

By the manner of Working of theſe Propoſitions *upon the* Projection, *the ingenious* Practitioner *may propoſe divers other* Queſtions *to himſelf, and* project *the* Sphere in Plano *ſutable unto them; in the* Practice *and true* performance *whereof he will accumulate to himſelf both* Pleaſure *and* Profit.

Beſides the working of theſe Propoſitions *upon the* Projection, *I have to every of them added the* Analogie *or* Proportion, *by which they may be reſolved by the* Canons *of Artificial* Sines *and* Tangents; *ſo that ſuch as require more exactneſs then can be expected from* Inſtrumental performance, *may by thoſe* Canons *work and reſolve their* Queſtions *more accurately. Many more* Propoſitions *I could have added, but theſe (for the* Aſtronomicall *part) I deemed ſufficient: ſuch as concern* Geographie *and* Navigation *ſhall follow in convenient place.*

Prop. I.

The diſtance of the Sun *from the neareſt* Æquinoctial Point (*either* Aries *or* Libra) *given, to find his* Declination.

IN the *Projection* this *Propoſition* is to be reſolved upon the *Right-angled Spherical Triangle* A *k* ♎, right-angled at ♎: which Triangle is conſtituted by the Interſection of the Arches of three great Circles of the Sphere; namely, of A ♎, *an Arch of the Ecliptick*; A *k*, *an Arch of the Æquinoctial*; and of ♎ *k*, *an Arch of a great Circle paſſing through* ♑ *and* ♋ *the Poles of the Ecliptick*, and cutting the *Ecliptick* at right Angles in ♎, which is the place of the *Sun* at the time of the *Queſtion.*

In which Triangle you have given (beſides the right Angle at

at ♎) (1.) the Side A ♎, 59 degr. which is the diſtance of the Sun from the neareſt Æquinoctial Point *Libra*; (2.) the Angle ♎ A *k*, 23 degr. 30 min. which is the Angle that the *Ecliptick* makes with the *Æquinoctial*, and is alwaies equal to the greateſt *Declination* of the Sun. Wherefore in this *Triangle* you having given the Baſe A ♎, and the Angle at the Baſe *k* A ♎, you may find the Perpendicular *k* ♎, the *Sun's Declination*, by the 11. Caſe of Right-angled Sphericall Triangles in the firſt Part of this Book. For which this is

The Analogie or Proportion.

As the Radius 90 degr. is to the Sine of the Sun's greateſt Declination 23 degr. 30 min.
So is the Sine of the Sun's diſtance from the next Æquinoctial Point *Libra* 59 degr. to the Sine of the Sun's preſent Declination 20 degr.

To reſolve the Triangle upon the Projection,

Lay a Ruler upon the Pole of the Circle R ♎ Q, (which is at ♉,) and the Point *k*, it will cut the Meridian in the Point *l*: Alſo lay a Ruler from ♉ to ♎, it will cut the Meridian in the Point ♈. So the diſtance *l* ♈, being meaſured upon your Line of Chords, will contain 20 degr. the *Sun's Declination*, he being in the 29. degree of ♎.

The like Declination *the* Sun *hath when he is in* 29 *degr. of* Taurus, *in* 1 *degr. of* Leo, *or* 29 *degr. of* Scorpio, *every of which Points are diſtant from one of the* Æquinoctial Points Aries *or* Libra 59 *degr.*

Prop. II.

The Latitude *of the Place, and the* Declination *of the Sun, being given, to find the* Ascensional Difference.

UPON the Projection this Proposition is to be resolved by finding the Side A B of the Right-angled Spherical Triangle A ⊙ B, Right-angled at B; which Triangle is compounded of three Arches of great Circles, namely, of A ⊙, an Arch of the Horizon, A B, an Arch of the Æquinoctial, and ⊙ B, an Arch of an Hour-Circle.

In this Triangle you have given (1.) the Side ⊙ B, the Sun's Declination 20 d. (2.) the Angle ⊙ A B, the Complement of the Latitude 38 d. 30 m. and the right Angle at B. In this Triangle therefore you have given ⊙ B, the Perpendicular, and ⊙ A B, the Angle at the Base, to find the Base A B, which you may doe by the 14. Case of Right-angled Spherical Triangles. For which this is

The Analogie or Proportion.

As the Co-tangent of the Latitude 38 degr. 30 min. is to the Tangent of the Sun's Declination 20 degr.
So is the Radius 90 degr. to the Sine of the Ascensional Difference 27 degr. 14 min.

To resolve the Triangle upon the Projection,

Lay a Ruler to P, the Pole of the World, (and also of the Æquinoctial,) and the Point B, it will cut the Meridian Circle in the Point *b*; the distance *b* S, being taken in your Compasses and measured upon your Line of Chords, will reach from the beginning thereof to 27 degr. 14 min. the Ascensional Difference; which is so much as the Sun riseth or setteth before or after Six a Clock. So these 27 degr. 14 min. being turned into

into Time (by allowing 15 deg. for one Hour, and one Degree for 4 minutes of Time) is 1 Hour and 49 min. and ſo much doth the Sun riſe or ſet before or after the hour of Six, according to the time or ſeaſon of the Year: for if the Sun hath *North Declination*, then he *riſeth before Six*, and *ſets after*; but if the Sun have *South Declination*, then doth he *riſe after*, and *ſet before* Six.

This Aſcenſional Difference *being added to 6 Hours will give you the* Semidiurnall Arch *or Half-length of the Day*; *and being taken from ſix Hours, will leave the* Seminocturnall Arch *or Half-length of the Night.*

The Semidiurnall Arch, *when the Sun hath* 20 *degr. of North Declination, is repreſented in the Projection by the Arch* ♈ *a* ☉, *and the* Seminocturnall Arch *by* ☉ ♎. *The* Semidiurnall Arch, *when the Sun hath* 20 *degr. of South Declination, is repreſented by the Arch* ♎ E r, *and the* Seminocturnall *by the Arch* r c ♈.

Though I have ſhewed how theſe may be found by adding *and* ſubtracting *the* Aſcenſionall Difference; *yet they may be found by the* Projection, *for the Arches are meaſured upon the* Æquinoctial. *Wherefore lay a Ruler to* P, *the Pole of the World, and the Point* B, *it will cut the Meridian Circle in* b: *So the diſtance* b Æ, *being meaſured by your Chord, will be* 117 *degr.* 14 *min. the* Semidiurnall Arch, *and* b ♈ *meaſured will be* 62 *degr.* 11 *min. for the* Seminocturnall Arch.

Prop. III.

The Latitude *of the Place, and the* Declination *of the Sun, being given, to find his* Amplitude.

THERE are two Triangles upon the Projection, by the reſolving of either the Propoſition may be reſolved, and both of them Right-angled. The one is the Triangle P☉O,

Right-angled at O. The other is the Triangle made use of in the last Proposition, A ⊙ B. The first Triangle is constituted of these three Arches, viz. P O, an Arch of the Meridian, P ⊙, an Arch of an Hour-Circle, and ⊙ O, an Arch of the Horizon.———The second Triangle consisteth of A ⊙, an Arch of the Horizon, ⊙ B, an Arch of an Hour-Circle, and A B, an Arch of the Æquinoctial. In the first Triangle you have given the Perpendicular P O, the Latitude of the Place, and the Hypotenuse P ⊙, the Complement of the Sun's Declination 70 degr. by which you are to find the Base ⊙ O, the Sun's Amplitude from the North part of the Meridian, which may be found by the 7. Case of Right-angled Spherical Triangles.—In the second Triangle B A ⊙ (which is that I will first exemplifie in) you have given (1.) the Perpendicular ⊙ B, the Sun's Declination 20 degr. (2.) the Angle at the Base ⊙ A B, the Complement of the Latitude 38 degr. 30 min. to find the Hypotenuse ⊙ A, the Sun's Amplitude from the East or West. So having the Perpendicular ⊙ B, and the Angle at the Base ⊙ A B, you may find the Hypotenuse ⊙ A by the 4. Case of Right-angled Spherical Triangles. And for it this is

The Analogie or Proportion.

As the Co-sine of the Latitude 38 degr. 30 min. is to the Radius 90 degr.
So is the Sine of the Sun's Declination 20 degr. to the Sine of the Amplitude from the East or West Points of the Horizon 33 degr. 20 min.

To work the Proposition upon the Projection,

Lay a Ruler upon Z the Zenith, (which is also one of the Poles of the Horizon,) and to the Point ⊙, it will cut the Meridian Circle in *d*; and laid from Z to A, it will cut the Nadir Point in N: So the distance N *d*, being taken in your Compasses and measured upon your Line of Chords, will contain

tain 33 degr. 20 min. the quantity of the Hypotenuſe, which is the Amplitude of the Sun's riſing or ſetting from the true Eaſt or Weſt Points of the Horizon.

To perform the ſame Work by the other Triangle P ⊙ O, Lay a Ruler to Z the Zenith, and the Point ⊙, and it will cut the Meridian Circle in the Point *d*, as before: So the diſtance *d* O, meaſured upon your Line of Chords, will contain 56 degr. 40 min. which is the Amplitude of the Sun's riſing or ſetting from the North Point of the Horizon.

PROP. IV.

The Latitude *of the Place, and the* Declination *of the Sun, being given, to find the* Angle of the Sun's Poſition *at the time of his* riſing.

THIS Propoſition is reſolvable upon the Triangle P ⊙ O: In which there is given (1.) the Hypotenuſe P ⊙, the Complement of the Sun's Declination; (2.) the Perpendicular P O, the Latitude: and it is required to find the Angle P ⊙ O: which may be found by the 15. Caſe of Right-angled Sphericall Triangles, and by the following

Analogie or Proportion.

As the Co-ſine of the Declination 70 degr. is to the Radius 90 degr.
So is the Sine of the Latitude 51 degr. 30 min. to the Sine of the Angle of the Sun's Poſition at the time of his riſing.

By the Projection.

Lay a Ruler to X, (the Pole of the Hour-Circle P ⊙ S,) and the Point ⊙; the Ruler ſo laid will cut the Meridian Circle near the Point ♎: then ſet 90 degr. from ♎ to *x* upon the Meridian, and lay the Ruler from X to *x*; ſo ſhall it cut the Hour-

Circle S ⊙ P (being continued without the Meridian Circle) in the Point ♦. Again, the Ruler laid from ⊙ to ♦ will cut the Meridian Circle in the Point 2. So the diſtance O 2, being taken in the Compaſſes and applied to the Line of Chords, will be found to contain 56 degr. 29 min. which is the quantity of the Angle P ⊙ O.

Prop. V.

The Sun's Declination, *and his* Amplitude *from the* North *part of the* Horizon, *being given, to find the* Latitude.

IN the ſame Triangle as in the laſt Propoſition P ⊙ O, you have given (1.) the Baſe ⊙ O, the Sun's Amplitude from the North part of the Horizon; (2.) the Hypotenuſe P ⊙, to find the Perpendicular P O. So there are two Sides given to find the third, which you may doe by the 8. Caſe of Right-angled Spherical Triangles.

The Analogie or Proportion.

As the Co-ſine of the Amplitude from the North 33 degr. 20 min. is to the Radius 90 degr.
So is the Sine of the Declination 20 degr. to the Co-ſine of the Latitude 38 degr. 30 min.

By the Projection.

This is eaſie; for if you take in your Compaſſes the diſtance from O to P upon the Meridian, and meaſure it upon your Line of Chords, it will be found to contain 51 degr. 30 min. the Latitude required.

Prop.

Prop. VI.

The Sun's greateſt Declination, *with his Diſtance from the next* Æquinoctial Point (Aries *or* Libra,) *being given, to find his* right Aſcenſion.

THE Triangle A *k* ♎ in the Projection reſolves this Propoſition, in which you have given (1.) the Side A ♎, the diſtance of the ☉ from ♎; (2.) the Angle ♎ A *k*, the Sun's greateſt Declination, and the right Angle at ♎: So you have given the Baſe, and the Angle at the Baſe to find the Hypotenuſe, which you may doe by the Caſe of Right-angled Sphericall Triangles, and this

Analogie or Proportion.

As the Radius 90 degr. is to the Co-ſine of the greateſt Declination 66 degr. 30 min.
So is the Tangent of the Sun's diſtance from the next Æquinoctial point *Libra* 59 d. to the Tangent of the right Aſcenſion 56 degr. 50 min.

By the Projection.

Lay a Ruler to P and *k*, it will cut the Meridian Circle in *s*: the diſtance S *s*, taken and meaſured upon your Line of Chords, will contain 56 degr. 50 min. the Sun's right Aſcenſion.

I ſhould here ſhew how the right Aſcenſion and Declination of a Star might be found; but, the Calculation Trigonometricall being very laborious, I have therefore in this place omitted it, becauſe I have in another Treatiſe, now ſpeedily to be publiſhed, framed Tables of the Longitude, Latitude, Right Aſcenſion, Declination,

clination, *and* Semidiurnall Arches, *of the most eminent Fixed Stars in the Heavens, exactly calculated for the Year of our Lord* 1670; *and also other Tables of the Sun's* right Ascension *for every Degree of the* Ecliptick *and Day of the Year: by help of which Tables, the Hour of the Night, the Rising, Setting, and the time of their coming to the South, may be obtained by the Rules and Directions prescribed in the forementioned Treatise, which will contain (besides the Tables here mentioned) all such others as at any time the Sea-man shall have occasion for in his Practice, and divers other things too tedious here to enumerate. But in the mean time I shall request the Reader to be satisfied with these two Tables following: These Exercises being things onely Practicall.*

A Table of the Sun's right Ascension for every Degree of the Ecliptick.

Degr.	♈		♉		♊		♋		♌		♍	
0	0	0	27	54	57	48	90	0	122	12	152	6
1	0	55	28	51	58	51	91	5	123	14	153	3
2	1	50	29	49	59	53	92	11	124	16	154	1
3	2	45	30	46	60	56	93	16	125	19	154	58
4	3	40	31	44	61	59	94	22	126	20	155	54
5	4	35	32	42	63	2	95	27	127	22	156	51
6	5	30	33	40	64	6	96	32	128	24	157	48
7	6	25	34	38	65	9	97	38	129	25	158	44
8	7	21	35	37	66	13	98	43	130	26	159	40
9	8	16	36	36	67	17	99	48	131	27	160	37
10	9	11	37	34	68	21	100	53	132	28	161	33
11	10	6	38	33	69	25	101	58	133	28	162	29
12	11	1	39	33	70	29	103	3	134	29	163	25
13	11	57	40	32	71	34	104	8	135	29	164	20
14	12	52	41	31	72	38	105	13	136	29	165	16
15	13	48	42	31	73	43	106	17	137	29	166	12
16	14	44	43	31	74	47	107	22	138	29	167	7
17	15	40	44	31	75	52	108	26	139	28	168	3
18	16	35	45	31	76	57	109	31	140	27	168	58
19	17	31	46	32	78	1	110	35	141	27	169	54
20	18	27	47	32	79	7	111	39	142	26	170	49
21	19	23	48	33	80	12	112	43	143	24	171	44
22	20	20	49	34	81	17	113	47	144	23	172	39
23	21	16	50	35	82	22	114	51	145	22	173	35
24	22	12	51	36	83	28	115	54	146	21	174	30
25	23	9	52	38	84	33	116	57	147	18	175	25
26	24	6	53	40	85	38	118	1	148	16	176	20
27	25	2	54	41	86	44	119	4	149	14	177	15
28	25	59	55	44	87	49	120	7	150	11	178	10
29	26	56	56	46	88	55	121	9	151	9	179	5
30	27	54	57	48	90	0	122	12	152	6	180	1

A Table of the Sun's right Ascension for every Degree of the Ecliptick.

Degr.	♎		♏		♐		♑		♒		♓	
0	180	0	207	54	237	48	270	0	302	12	332	6
1	180	55	208	51	238	51	271	5	303	14	333	3
2	181	50	209	49	239	53	272	11	304	16	334	1
3	182	45	210	46	240	56	273	16	305	19	334	58
4	183	40	211	44	241	59	274	22	306	20	335	54
5	184	35	212	42	243	2	275	27	307	22	336	51
6	185	30	213	40	244	6	276	32	308	24	337	48
7	186	25	214	38	245	9	277	38	309	25	338	44
8	187	21	215	37	246	13	278	43	310	26	339	40
9	188	16	216	36	247	17	279	48	311	27	340	37
10	189	11	217	34	248	21	280	53	312	28	341	33
11	190	6	218	33	249	25	281	58	313	28	342	29
12	191	2	219	33	250	29	283	3	314	29	343	25
13	191	57	220	32	251	34	285	8	315	29	344	20
14	192	52	221	31	252	38	286	13	316	29	345	16
15	193	48	222	31	253	43	287	17	317	29	346	12
16	194	44	223	31	254	47	288	22	318	29	347	7
17	195	40	224	31	255	52	289	26	319	28	348	3
18	196	35	225	31	256	57	290	31	320	27	348	58
19	197	31	226	32	258	1	291	35	321	27	349	54
20	198	27	227	32	259	7	292	39	322	26	350	49
21	199	23	228	33	260	12	293	43	323	24	351	44
22	200	20	229	34	261	17	294	47	324	23	352	39
23	201	16	230	35	262	22	295	51	325	22	353	35
24	202	12	231	36	263	28	296	54	326	21	354	30
25	203	9	232	38	264	33	297	57	327	18	355	25
26	204	6	233	40	265	38	298	1	328	16	356	20
27	205	2	234	41	266	44	299	4	329	14	357	15
28	205	59	235	44	267	49	300	7	330	11	358	10
29	206	56	236	46	268	55	301	9	331	9	359	5
30	207	54	237	48	270	0	302	12	332	6	360	0

The right Ascension, Declination, and Magnitude of some principal Fixed Stars.

The Stars Names.	Right Ascension.	Declination.		Magnitude.
	D. M.	D. M.		
The Pole-Star	7 47	87 27	N	2
The Girdle of *Andromeda*	12 32	33 48	N	2
The former Horn of the Ram	23 38	17 37	N	4
Bright Star in the Ram's Head	26 56	21 48	N	3
The Whale's Jaw	41 3	2 42	N	2
The Head of *Medusa*	41 27	39 35	N	3
The Bull's Eye	64 0	15 46	N	1
The Goat	72 44	45 35	N	1
The former Shoulder of *Orion*	76 38	4 59	N	2
The latter Shoulder of *Orion*	84 7	7 18	N	2
The great Dog	97 27	16 13	S	1
The uppermost Head of the Twins	108 1	32 35	N	2
The little Dog	110 17	6 6	N	2
The lower Head of the Twins	111 0	28 49	N	2
The Crib	125 4	20 52	N	Neb.
Hydra's Heart	137 39	7 10	S	2
Lion's Heart	147 27	13 39	N	1
Lion's Loins	163 54	22 26	N	2
Lion's Tail	172 49	16 32	N	1
The Virgin's Girdle	189 32	5 20	N	3

The right Ascension, Declination, and Magnitude of some principal Fixed Stars.

The Stars Names.	Right Ascension.		Declination.			Magnitude.
	D.	M.	D.	M.		
Aliot	189	36	57	36	N	2
Vindemiatrix	191	15	12	51	N	3
The Virgin's Spike	196	44	9	17	S	1
Arcturus	209	56	21	4	N	1
The Southern Balance	217	56	14	32	S	2
The Northern Balance	224	31	8	2	S	2
In the Serpent's Neck	231	49	7	35	N	3
The Scorpion's Heart	242	4	25	34	S	1
Hercules Head	254	40	14	51	N	3
Ophinchus Head	259	41	12	52	N	3
The Harp	276	17	38	30	N	1
The Vulture	293	27	8	1	N	2
The upper Horn of the Goat	299	30	13	32	S	3
Left Hand of *Aquarius*	307	10	10	43	S	4
Left Shoulder of *Aquarius*	318	18	7	2	S	3
Pegasus Mouth	321	49	8	18	N	3
Right Shoulder of *Aquarius*	326	59	1	58	S	3
Fomahant	339	29	31	23	S	1
Upper Wing of *Pegasus*	341	53	13	21	N	2
In the tip of *Pegasus* Wing	358	52	13	15	N	2

PROP. VII.

The Latitude *of the Place and the Sun's* Declination *being given, to find at what Hour the Sun will be upon the true* Eaſt *or* Weſt *Points.*

UPON the Projection there are two Right-angled Sphericall Triangles, by either of which this Propoſition may be ſolved. The one is the Triangle Z P *o*, made by the Interſections of Z *o*, an Arch of the Prime Verticall, P *o*, an Arch of an Hour-Circle, and Z P, an Arch of the Meridian. In which Triangle there is given Z P, the Perpendicular, the Complement of the Latitude of the Place 38 degr. 30 min. and the Hypotenuſe P *o*, the Complement of the Sun's Declination 70 degr. to find the Angle at the Perpendicular Z P *o*, which you may doe by the 14. Caſe of Right-angled Sphericall Triangles.

The other Triangle is *o* C A, right-angled at C, and is conſtituted of *o* C, an Arch of an Hour-Circle, C A, an Arch of the Æquinoctial, and *o* A, an Arch of the Prime Verticall. In which Triangle you have given, (1.) the Perpendicular, O C, the Sun's Declination; (2.) the Angle at the Baſe, C A *o*, the Latitude 51 degr. 30 min. to find the Baſe C A. Thus having the Perpendicular and the Angle at the Baſe, you may find the Baſe C A as followeth, this being

The Analogie or Proportion.

As the Tangent of the Latitude 51 degr. 30 min. is to the Tangent of the Sun's Declination 20 degr.
So is the Radius 90 degr. to the Co-ſine of the Hour from Noon.

To resolve the Proposition by the Projection,

Lay a Ruler upon P, the Pole of the World, and the Angle C of your Triangle, the Ruler will cut the Meridian Circle in the Point *g*: So *g* Æ, being taken in your Compasses and measured upon your Line of Chords, will be found to contain 73 degr. 10 min. which converted into Hours and Minutes will be 4 hours and about 53 min. So that the Sun, when he hath 20 degr. of Declination, will come to the East Point at 7 min. past 7 in the Morning, and will be due West 53 min. after 4 in the Afternoon.

Prop. VIII.

Having the Latitude *of the Place, and the Sun's* Declination, *given, to find what* Altitude *the Sun shall have when he is upon the true* East *or* West *Points.*

THIS Proposition may be resolved by either or both of the Triangles mentioned in the last Proposition. For in the Triangle Z P *o* you have given P Z, the Perpendicular, and P O, the Hypotenuse, to find Z *o*, the Base, by the 8. Case of Right-angled Spherical Triangles.

But in the Triangle *o* C A, you have given (1.) the Perpendicular *o* C, the Sun's Declination 20 degr. (2.) the Angle at the Base *o* A C, the Latitude of the Place 51 degr. 30 m. to find the Hypotenuse *o* A, for which this is

The Analogie or Proportion.

As the Sine of the Latitude 51 degr. 30 min. is to the Sine of the Declination 20 degr.
So is the Radius 90 degr. to the Sine of the Sun's Altitude being due East or West 25 degr. 55 min.

To reſolve the Propoſition by the Projection,

Lay a Ruler upon O, (one of the Poles of the Prime Vertical) and to the Angle *o* of the Triangle; a Ruler thus laid will cut the Meridian Circle in the Point *p*: So the diſtance H *p*, being taken and meaſured upon the Line of Chords, will be 25 degr. 55 min. and ſuch height will the Sun have when he is either Eaſt or Weſt.

Prop. IX.

The Latitude *of the Place, and the* Declination *of the Sun, being given, to find what* Altitude *the Sun ſhall have at* Six *of the Clock.*

FOR finding of the Triangles upon the Projection, which will reſolve this and the following Propoſitions, you muſt ſuppoſe another Azimuth Circle to be drawn in the Projection from Z to N, and through that Point where the Parallel of Declination ♈ ☉ ♎, and the Axis of the World, or Hour-Circle of Six, P A S, do croſs each other. The drawing of which Azimuth Circle I purpoſely omitted, chiefly becauſe the Scheme in that place is more cumbred with Lines and Letters then any other part thereof: But you may well enough, for the ſolving of theſe two Propoſitions, imagine it to be drawn, the Pole whereof is at *. This Azimuth Circle being ſuppoſed to be drawn, you have upon the Projection two Triangles like-angled, which will perform the Work of reſolving this Propoſition. In one of which you have given the Baſe, which is the Complement of the Declination, and the Perpendicular, which is the Complement of the Latitude, to find the Hypotenuſe, which is the Complement of the Sun's Altitude required. This Triangle may be reſolved by the firſt Caſe aforegoing.——In the other Triangle there will be given the

Hypotenuſe,

Hypotenuse, which is the Sun's Declination, and the Angle at the Base, which is the Latitude, to find the Perpendicular, which is the Sun's Altitude at Six a Clock: To find which this is

The Analogie or Proportion.

As the Radius 90 degr. is to the Sine of the Sun's Declination 20 degr.
So is the Sine of the Latitude 51 degr. 30 min. to the Sine of the Sun's Altitude at Six 15 degr. 30 min.

To resolve the Proposition by the Projection,

Lay a Ruler upon the Point *, and that Point where the Parallel of Declination ☊⊙♌ crosseth the Axis or Hour of Six; the Ruler thus laid will cut the Meridian Circle in the Point *g*. So O *g*, being measured upon the Chords, will give you 15 degr. 30 min. And such Altitude will the Sun have at the Hour of Six in the Latitude of 51 degr. 30 min. when he hath 20 degr. of Declination.

Prop. X.

The Latitude *of the Place and the* Declination *of the Sun being given, to find what* Azimuth *the Sun shall have at* Six *a Clock.*

THE two Triangles that were supposed in the last Proposition to be drawn upon the Projection will resolve this Proposition also; but seeing the Triangles are not drawn, but supposed, I will onely give you the Analogie, and then the way of working it upon the Projection.

The Analogie or Proportion.

As the Co-sine of the Latitude 38 degr. 30 min. is to the Radius 90 degr.

So

So is the Co-tangent of the Sun's Declination 70 degr. to the Tangent of the Sun's Azimuth counted from the North part of the Meridian 77 degr. 14 min.

To reſolve the Propoſition upon the Projection,

Lay a Ruler to the Zenith-point Z, and upon the Point where the Parallel of Declination cuts the Hour-Circle of Six; the Ruler thus laid will cut the Meridian Circle in *r*: So the diſtance *r* O, being meaſured upon the Line of Chords, will contain 77 degr. 14 min. which is the Azimuth from the North part of the Meridian.——The diſtance N *r*, meaſured upon the Chords, will give you 12 degr. 46 min. which is the Azimuth from East or Weſt.——And *r* H, meaſured upon your Chord, will contain 102 degr. 46 min. his Azimuth from the South.

Prop. XI.

The Latitude *of the Place, the* Declination *of the Sun, and the Sun's* Altitude, *being given, to find the Sun's* Azimuth *either from the* East, North *or* South *Points of the* Horizon.

ALL the foregoing Propoſitions have been performed by the reſolving of a Right-angled Spherical Triangle: This and ſome others following require the reſolving of an Oblique-angled Triangle for their Solution. So this Propoſition is performed upon the Oblique-angled Triangle Z E P, which in the Projection is conſtituted by the Interſection of P E S, an Hour-Circle, Z E N, an Azimuth Circle, and Z H N O, the Meridian: and the Arches of theſe Circles interſecting each other in the Points Z, E, and P, do make the Triangle Z E P; in which you have given the three Sides, (1.) Z P, the Complement of the Latitude 38 degr. 30 min.

30 min. of the Place, (2.) P E, the Complement of the Sun's Declination South 110 degr. (3.) Z E, the Complement of the Sun's Altitude 78 d. to find the Angle E Z P, which may be reſolved by the 11. Caſe of Oblique-angled Spherical Triangles. But this being to reſolve a particular Propoſition, I ſhall give another Analogie or Proportion whereby to work it by the Canon.

Adde all the three Sides together into one Sum, and take the half thereof, from which half Sum ſubtract the Side P E, noting the difference, as here you ſee done.

		d.	m.
The Side	Z P	38	30
	P E	110	00
	Z E	78	00
The Sum		226	30
The half Sum		113	15
The difference between the half Sum and P E		3	15

Having found the Sum, the half Sum, and the difference, you may work by the following

Analogie or Proportion.

(1.) As the Radius 90 degr. is to the Co-ſine of the Altitude 78 degr.

So is the Co-ſine of the Latitude 38 degr. 30 min. to the Sine of a fourth number, which is 37 degr. 30 min.

(2.) As the Sine of the fourth number 37 degr. 30 min. is to the Sine of the half Sum 113 degr. 15 min.

So is the Sine of the difference 3 degr. 15 min. to another Sine, *viz.* 4 degr. 54 min. Unto which ſeventh Sine if you adde the Sine of 90 degr. half that Sum ſhall be the Sine of an Arch, whoſe Complement being doubled is the Azimuth from the North.

To

To reſolve this Probleme by the Projection.

That which is moſt intricate and difficult to perform by Numbers, is by Projection effected with the ſame eaſe as any of the reſt. As in this Propoſition, it is the Angle E Z P which is required.——Lay a Ruler upon the Zenith-point Z, and to the Point G, upon the Horizon; the Ruler thus laid will cut the Meridian Circle in the Point *g*. So the diſtance *g* O, being taken in your Compaſſes and meaſured upon your Line of Chords, will be found to contain 146 degr. which is the Sun's Azimuth from O, the North part of the Meridian.——But if you meaſure the diſtance between the *g* and H, it will contain 34 degr. which is the Azimuth from H, the South part of the Meridian.——And if you meaſure the diſtance *g* N upon your Chord-Line, you ſhall find that to contain 56 degr. and ſo much is the Sun's Azimuth from A, the Eaſt and Weſt Points of the Horizon.

This Example of finding the Azimuth was taken when the Sun had 20 *degr. of South Declination. I will now further exemplifie this Propoſition by finding the Azimuth when the Sun hath North Declination.——As let the Latitude be as before* 51 *d.* 30 *min. the Sun's Altitude* 12 *degr. and the Declination* 20 *d. North.*

To work this by the Canon of Sines differeth nothing from the former, for the Analogie or Proportion is general in all Caſes.

Upon the Projection it is reſolved (though the ſame way, yet) upon another Triangle, namely, the Triangle Z P *a*, in which is given (1.) Z P, the Complement of the Latitude 38 d. 30 min. (2.) Z *a*, the Complement of the Altitude 78 degr. (3.) the Complement of the Sun's Declination North 70 degr. and you are to find the Angle P Z *a*, the Sun's Azimuth from the North.

Lay a Ruler upon Z unto the Point *a*, it will cut the Meridian Circle in the Point *s*; the distance *s* O, being taken in your Compasses and applied to your Line of Chords, will there give you 72 degr. 52 m. And such is the Sun's Azimuth from the North.

If you subtract this Azimuth from the North 72 *degr.* 52 *m. from* 180 *degr. the remainer* 107 *degr.* 8 *min. will give you the Azimuth from the South, which upon the Projection is the distance* s H. —*And if from this Azimuth from the South* 107 *d.* 8 *min. you take* 90 *degr. the remainer* 17 *degr.* 8 *min. is the Azimuth from the East or West, which in the Projection is the distance* N s.

Prop. XII.

The Latitude *of the Place, the Sun's* Declination, *and the Sun's* Altitude, *being given, to find the* Hour *of the* Day.

THIS Propositon is performed by the resolving of the Oblique-angled Sphericall Triangle Z P *a*, composed of Z P, an Arch of the Meridian, Z *a*, an Arch of an Azimuth Circle, and of P *a*, the Arch of an Hour-Circle: In which you have given (as in the last Proposition) the three Sides, to find the Angle Z P *a*, which you may doe by the 11. Case of Oblique Sphericall Triangles.

To resolve this Proposition by the Canon; Adde the three Sides together, and from the half Sum of them subtract the Complement of the Sun's Altitude, and note the difference, as you see here done.

The

		d.	m.
The Side	Z P, the Complement of the Latitude	38	30
	Z *a*, the Complement of the Altitude	78	00
	P *a*, the Complement of the Declination	70	00
	The Sum	186	30
	The half Sum	93	15
	The difference between the half Sum and Z *a*, the Complement of the Altitude	15	15

Being thus prepared, you may reſolve the Propoſition by the Canon of Sines, by this

Analogie or Proportion.

(1.) As the Radius 90 degr. is to the Co-ſine of the Sun's Altitude 78 degr.

So is the Co-ſine of the Latitude 38 degr. 30 min. to a fourth Sine, viz. 35 degr. 48 min.

(2.) As this fourth Sine of 35 degr. 48 min. is to the Sine of the half Sum 93 degr. 15 min.

So is the Sine of the Difference 15 degr. 15 min. to another Sine, viz. to the Sine of 26 degr. 40 min. Unto which Sine if you adde the Sine of 90 degr. (or Radius,) half that Sum ſhall be the Sine of an Arch, whoſe Complement being doubled is the Hour from the Meridian 95 degr. 52 min.

To reſolve the Propoſition by the Projection.

In the Triangle Z P *a*, it is the Angle at P that is to be found. Wherefore lay a Ruler from the Point P to the Point *a*, and it will cut the Meridian Circle in *t*: So the Arch *t* Æ, being meaſured upon your Line of Chords, will be found to contain 95 d. 52 min. which is the Hour from the Meridian; and the Arch *t a*, being meaſured, will contain 84 degr. 8 min. which is the

Hour from Midnight. Also the Arch t S, being measured upon the Chord, will contain 5 degr. 52 min. the Hour from Six.

		d.	m.		hours	m.
The Arch	t Æ	95	52	converted into Time is	6	23
	t æ	84	08		5	36
	t S	05	52		0	23

To convert Degrees and Minutes of the Æquinoctial into Hours and Minutes of Time: Note that 15 *Degrees of the Æquinoctial make one Hour of Time, and one Degree* 4 *Minutes of Time. Therefore divide the Degrees of the Æquinoctial by* 15, *the Quotient is Hours; and multiply the Degrees by* 4, *and the Product will be Minutes of Time.——So the Hour from the Meridian being* 95 *degr.* 52 *min. divide* 95 *by* 15, *the Quotient is* 6 *Hours, and* 5 *remaining, which* 5 *multiply by* 4, *and it makes* 20 *Minutes of Time, and the* 52 *min. make* 3 *minutes of Time and more, almost* 4 *minutes. So that* 95 *degr.* 52 *min. of the Æquinoctial do make in Time* 6 *hours and almost* 24 *minutes.*

Prop. XIII.

The Declination, Altitude, *and* Azimuth *of the Sun, being given, to find the* Hour *of the Day.*

THE Triangle Z E P in the Projection resolves this Problem: in which there is given (1.) E P, the Complement of the Sun's Declination 70 degr. South; (2.) the Side E Z, the Complement of the Sun's Altitude 78 degr. and (3.) the Angle E Z P, the Sun's Azimuth from the North. So that you have two Sides and an Angle opposite to one of them given, to find the Angle opposite to the other, which you may doe by the 8. Case of Oblique-angled Spherical Triangles, and by the following

Ana-

Analogie or Proportion.

As the Co-ſine of the Declination 70 degr. is to the Sine of the Azimuth 146 degr. or 34 degr.
So is the Co-ſine of the Altitude 78 degr. to the Sine of the Hour from Noon 35 degr. 36 min.

By the Projection.

Lay a Ruler to the Pole P, and upon the Point D in the Æquinoctial; a Ruler thus laid will cut the Meridian Circle in *e*; the diſtance *e* Æ, being meaſured upon the Line of Chords, will give you 35 degr. 36 min. the Hour from Noon, which in Time is 2 hours and 22 minutes.

PROP. XIV.

The Sun's Declination, *his* Altitude, *and the* Hour *from Noon, being given, to find the Sun's* Azimuth *from the North part of the Meridian.*

IN the ſame Triangle Z E P you have given (1.) the Side E P, the Complement of the Sun's Declination South 70 d. (2.) the Side Z E, the Complement of the Altitude 78 degr. and (3.) the Angle Z P E, the Hour from Noon 36 degr. 35 m. That is, you have given (as before) two Sides and an Angle oppoſite to one of them, to find the Angle oppoſite to the other, which you may doe by the 8. Caſe of Oblique Spherical Triangles; or by this

Analogie or Proportion.

As the Co-ſine of the Altitude 78 degr. is to the Sine of the Hour from Noon 36 degr. 35 min.
So is the Co-ſine of the Sun's Declination 70 d. to the Sine of the Azimuth from the North part of the Meridian 146 degr. or 34 degr. from the South.

By

By the Projection.

Lay a Ruler to Z, and upon the Point G, which will cut the Meridian Circle in *g*: So *g* H, being meaſured upon your Line of Chords, will be found to contain 34 degr. the Azimuth from the South part of the Meridian, which being taken from 180 degr. the remainer will be 146 degr. equal to the Arch *g* O, the Azimuth from the North part of the Meridian.

Prop. XV.

The Hour *from Noon, the* Latitude *of the Place, and the* Altitude *of the Sun, being given, to find the* Angle *of the* Sun's Poſition.

IN the Oblique-angled Spherical Triangle Z P E you have given the Side Z P, the Latitude, Z E, the Complement of the Sun's Altitude, and Z P E, the Hour from Noon, to find the Angle Z E P, which is the Angle of the Sun's Poſition at the time of the Queſtion: So in the Triangle Z P E you have two Sides, with an Angle oppoſite to one of them, given, to find the Angle oppoſite to the other Side, which you may find by the 2. Caſe of Oblique-angled Spherical Triangles: For which this is

The Analogie or Proportion.

As the Co-ſine of the Sun's Altitude 78 degr. is to the Sine of the Hour from Noon 35 degr. 36 min.

So is the Co-ſine of the Latitude 38 degr. 30 min. to the Sine of the Angle of the Sun's Poſition at the time of the Queſtion 21 degr. 45 min.

By the Projection.

This is the moſt troubleſome Propoſition that we have yet met

met withall to be reſolved by the Projection; and yet it is alſo thereby eaſily reſolved in this manner.

Take in your Compaſſes 90 degr. of your Chords; then lay a Ruler upon Y, (the Pole of the Hour-Circle P E S,) and the angular Point E; it being ſo laid will cut the Meridian Circle in *v*. Then ſet 90 degr. of your Line of Chords from *v* to *x* upon the Meridian Circle, and the Ruler laid from Y to *x* will cut the Hour-Circle P E S in the Point *y*.

Again, lay a Ruler to ⊙, (the Pole of the Azimuth Circle Z E N,) and to the angular Point E; it being ſo laid will cut the Meridian Circle in the Point M. Set 90 degr. from M to *z* upon the Meridian Circle, and lay a Ruler upon ⊙ and *z*; it will cut the Azimuth Circle (it being continued without the Meridian Circle) in the Point ♪.

Laſtly, Lay a Ruler to the angular Point E, and this Point ♪, it will cut the Meridian Circle in ♑; alſo lay a Ruler from E to *y*, it will cut the Meridian in ♌. The diſtance ♑ ♌, being taken in the Compaſſes and meaſured upon your Line of Chords, will contain 21 degr. 45 min. and that is the quantity of the enquired Angle Z E P, which is the Angle of the Sun's Poſition at the time of the Queſtion.

Prop. XVI.

The Sun's Altitude, *his* Declination, *and* Azimuth *from the* North, *being given, to find the* Latitude.

IN the former Triangle Z E P, let there be given (1.) E Z, the Complement of the Sun's Altitude 78 degr. (2.) E P, the Complement of the Sun's Declination (or his diſtance from the North-pole) 110 degr. (3.) the Angle E Z P, the Azimuth from the North.——By the laſt Propoſition find the Angle of the Sun's Poſition, Z E P, which had, the Side Z E, the Complement of the Latitude, may be found by this

Analogie or Proportion.

As the Sine of the Sun's Azimuth 146 degr. (or 34 degr.) is to the Sine of the Sun's diſtance from the North-pole 110 d. (or 70 degr.)
So is the Sine of the Angle of the Sun's Poſition 21 degr. 45 m. to the Complement of the Latitude 38 degr. 30 min.

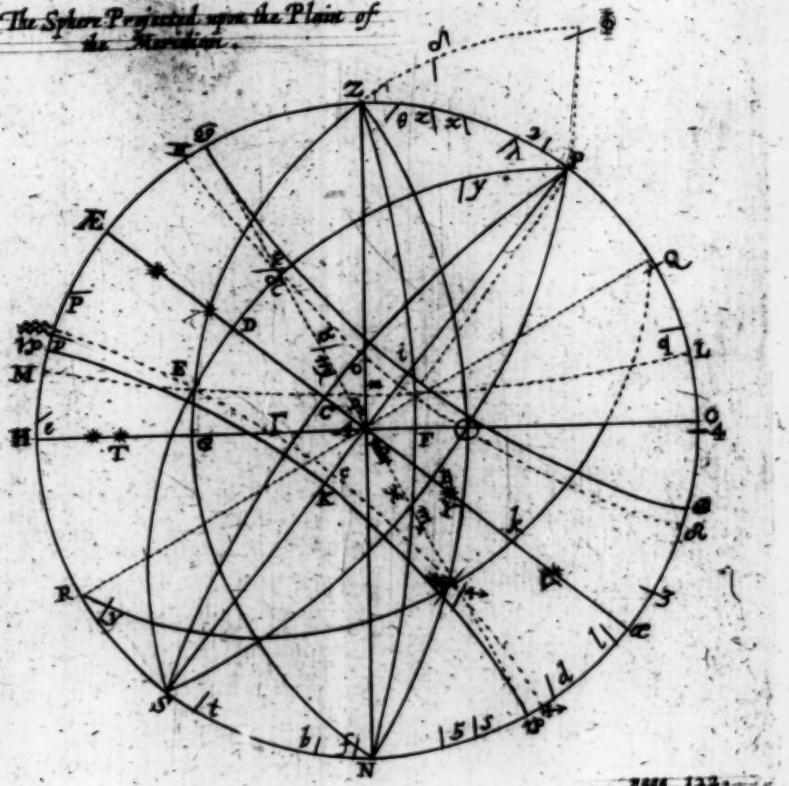
By the Projection.

It being the Side Z P that is required, foraſmuch as that is an Arch of the Meridian Circle, you have no more to doe but to take the diſtance Z P in your Compaſſes, and meaſure it upon your Line of Chords; which being done, you will find it will contain 38 degr. 30 m. the Complement of the Latitude, which taken from 90 degr. leaves 51 degr. 30 m. the Latitude it ſelf.

By the Solution of the foregoing Propoſitions, both by Trigonometricall Calculation by the Canons of Sines and Tangents, and alſo by the Projecting of the Sphere, you may eaſily diſcern the facility of the one above the other, namely, that of Projection; foraſmuch as thoſe Propoſitions which by Calculation are moſt troubleſome, are by the Projective way moſt eaſie.——For, here is no need of letting fall Perpendiculars to reduce the Oblique Triangle into two Right-angled Triangles, and ſo making of two works for ſolving of one Problem. Beſides, the Projection renders every Triangle ſo naturally to the eye, that they are reſolved (as it were) by looking on them.——Many more Examples I might have given upon this one Projection, but I ſee the Scheme grow too full of Lines and Letters, that makes me here break off. And now I will ſhew how the ingenious young Seaman may apply theſe Propoſitions to his uſe at Sea.

The

The Sphere Projected upon the Plaine of the Meridian.



page 122.

The foregoing PROPOSITIONS applied to Practice:

By which the Ingenious young Sea-man may make them ſerviceable to him at Sea, to ſeverall good and uſefull Purpoſes.

The Seventh EXERCISE.

OF *the foregoing* Aſtronomicall Propoſitions *the ingenious* Sea-man *may make good and profitable uſe at Sea. For ſome of them will be aſſiſtent to him in the finding of the* Latitude *of the Place he is in.*——*Some of them will help him to find* the time of the Sun's riſing and ſetting *in any place, and at any time of the Year.*——*Some will help him to the* Hour *of the* Day,——*Some to the* Hour *of the* Night, *at any time and in any place.*——*And divers of them to find the* Variation of the Compaſs. *Examples of all which I will inſtance in, ſo that he may put them in practice at Sea.*

I. *Propositions assistent to find the Latitude.*

THE *Propositions* which may be applied to the finding of the *Latitude* are the First and the Sixteenth.

The first *Proposition* is to find *The Sun's Declination*, which being obtained, and the Sun's *Meridian Altitude* observed at *Sea* or *Land* in any part of the World, the *Latitude* of that Place, by help of them, may be known; in which there are severall *Cases*, according as the Sun hath either *North* or *South Declination*, and as the *Sun* is situate, he being either upon the *North* or *South-side* of the *Meridian*.—The severall Varieties are these which follow.

When the Sun is in the *Æquinoctial*, having no *Declination*, and the *Meridian Altitude* is observed on the

- SOUTH-side of the *Meridian*, — The *Meridian Altitude* taken from 90 degr. leaves the Elevation of the *North-pole*.
- NORTH-side of the *Meridian*, — The *Meridian Altitude* taken from 90 degr. leaves the Elevation of the *South-pole*.

When the Sun's *Declination* is

- NORTH, — If the *Meridian Altitude* be less then 90 d. and the Sun upon the *South-side* of the *Meridian*; the *Sun's Declination*, being taken from the *Meridian Altitude*, leaves the height of the *Æquinoctial*, which taken from 90 d. gives the *Latitude North*.
- SOUTH, — If the *Meridian Altitude* be less then 90 d. and the Sun upon the *South-side* of the *Meridian*, adde the *Meridian Altitude* and *Declination* together; their Sum is the height of the *Æquinoctial*, which taken from 90 degr. leaves the *Latitude North*. But if the Sum of the *Declination* and *Altitude* exceed 90 degr. take 90 degr. therefrom, the remainer is the *Latitude South*.

When

When the Sun's *Declination* is

NORTH. If the *Meridian Altitude* be leſs then 90 d. and the Sun upon the *North-ſide* of the *Meridian*, adde the *Altitude* and *Declination* together; their Sum is the height of the *Æquinoctial*, which taken from 90 d. leaves the *Latitude South*.—But if the Sum be above 90 d. take 90 d. therefrom, the remainer is the *Latitude North*.

SOUTH. If the *Meridian Altitude* be leſs then 90 d. and the Sun upon the *North-ſide* of the *Meridian*, ſubtract the *Declination* from the *Meridian Altitude*; the remainer is the height of the *Æquinoctial*, which taken from 90 degr. leaves the *Latitude South*.

When the Sun's *Declination* is NORTH, SOUTH, } If the *Meridian Altitude* be juſt 90 degr. the *Sun's Declination* is the *Latitude* { *North*. *South*.

If the *Meridian Altitude* be obſerved under the *Pole*, within the bounds of the *Polar Circles*, in ſuch caſe the *Sun's Declination* muſt be taken from 90 degr. and what remains is his diſtance from the *Pole*; which being added to the *Meridian Altitude*, the Sum is the *Latitude* of the Place.

The other Propoſition which will be aſſiſtent to find the *Latitude* is the Sixteenth. If you ſet your *Compaſs* to the given *Azimuth*, and when the Sun is upon that *Azimuth*, if you take his height, you may find your *Latitude* either by *Trigonometricall Calculation* or by *Projection*. As in the Sixteenth Propoſition.

II. *Propoſitions aſſiſtent to find the time of the Sun's* riſing *and* ſetting.

THE principal Propoſition for this purpoſe is the Second, which is to find the *Aſcenſionall Difference*, from which the time of the *Sun's riſing* and *ſetting*, the *Semidiurnall* and *Seminoctiurnall Arches*, may be gathered; and from thence the length of the *Day* and *Night*: all which are plainly ſhewed in the Propoſition it ſelf.

III. *Propoſitions aſſiſtent to find the* Hour *of the* Day *and* Night.

THE Twelfth and Thirteenth Propoſitions will be ſerviceable to find the Hour of the Day. The Twelfth giving the Hour at any time of the Day by the Work of that Propoſition it ſelf.——The Thirteenth findeth the *Hour* upon a given *Azimuth* and *Altitude*. Wherefore ſet your Compaſs to the given *Azimuth*, and obſerve his *Altitude* when he cometh upon that *Azimuth*; the Sun's *Declination* (or time of the year) being known, you may then find the *Hour* by the Work of the Thirteenth Propoſition.

The Propoſition that will be aſſiſtent to you in finding the *Hour* of the *Night* is chiefly the Sixth, it ſhewing how to find the Sun's *right Aſcenſion*, which, with the aſſiſtence of thoſe other Tables which follow that *Propoſition*, will help you to the *Hour* of the *Night*, and alſo to find at what time any of the *Stars* there inſerted in the *Table* will be upon the *Meridian*. The manner how to effect either ſhall be ſhewed in theſe two following Problems.

PROBL.

PROBL. I.

How to find at what time any of the Stars *in the Table of the Sixth* Proposition *will be upon the* Meridian.

SUbstract the *Right Ascension* of the Sun from the *Right Ascension* of the *Star*, the remainer is the time of the *Star's* coming to the *Meridian* after noon. But if the *right Ascension* of the *Star* be less then the *Right Ascension* of the *Sun*, adde 360 degr. thereto, and substract the *Right Ascension* of the *Sun* from the Sum, and the Remainer is the time of the *Star's* coming to the *Meridian*.

Example. Upon the fourth of *October* 1667, the Sun being in 21. degr. of *Libra*, I would know at what time *Sirius* (or the *Great Dog*) will be upon the *Meridian*.

	d.	m.
The *Right Ascension* of *Sirius* is	97	27
The *Right Ascension* of the *Sun* that day is	199	23
Because Substraction cannot be made, adde 360 d. to the *Right Ascension* of the *Star*	360	00
The Sum is	457	27
The *Sun's Right Ascension* substracted from 458 degr. 4 min. leaves the time of the *Star's* coming to the *Meridian*	258	04

Which 258 degr. 4 min. being converted into Time make 17 hours almost, that is, at 5 of the Clock the next Morning, *Sirius* will be upon the *Meridian*.

PROBL.

Probl. II.

To find the Hour *of the* Night *by any of the* Stars *that are in the* Table *of the Sixth* Proposition.

TAKE the *Altitude* of the *Star*, and by his *Declination* and *Altitude* find the Hour (by the Twelfth Proposition) as if it were by the Sun, which I call the *Star's Hour*. Then comparing the *Right Ascension* of the *Sun* with the *Right Ascension* of the *Star*, you may come to find the *Hour* of the *Night*. *Example.* Upon the 16. day of *November*, in the Morning, I took the *Altitude* of *Arcturus*, finding it to be 27 degr. 12 m. and his *Declination* (by the Table) I find to be 20 degr. 58 m. By help of these two and the *Latitude* I find the *Star's Hour* to be 72 degr. Then compare the *Sun's Right Ascension* with the *Star's Right Ascension*, and find his time of coming to the *Meridian*, as in the former *Probl.* the difference between the *Star's Hour* and his coming to the *Meridian* is the *Hour* of the *Night*. See the manner of the Operation.

	d.	m.
The *Right Ascension* of *Arcturus*	210	13
The *Right Ascension* of the *Sun*	242	00
Adde to make Subt.	360	00
The Sum is	570	13
The *Sun's Right Ascension* substracted, rests	328	13
From which take 180 degr. or 12 hours	180	00
Rests	148	13
The *Star's Hour* substracted	72	00
Leaves the *Hour* of the *Night*	76	13

Which converted into Time is 5 h. 5 m. and that is the Hour in the Morning.

IV *Pro-*

IV. *Propoſitions aſſiſtent to the finding of the* Variation of the Compaſs.

THE Propoſitions that will be ſerviceable herein are the 3.7.8.9. 10. 11. and 14. but more eſpecially the *Third* and the *Eleventh:* and thoſe I ſhall here illuſtrate by *Example*, though all the reſt (as occaſion may fall out) will be alſo uſefull thereunto. By the Third Propoſition you may find the Amplitude of the Sun's Riſing and Setting.———By the Eleventh you may find the Azimuth at any time of the day.— By either of which the *Variation of the Compaſs* may be found, and alſo which way it varieth.

I. *To find the Variation by the* Amplitude.

BY the Third Propoſition you found the Sun's *Amplitude* at his *riſing* or *ſetting* to be 33 degr. 20 min. from the true *Eaſt* or *Weſt* Points of the *Horizon* towards the *North*. Having thus before-hand found the *Amplitude*, in the Morning I ſet my *Compaſs* to the Sun at his Riſing; and if I find that the Sun by my Compaſs do riſe 33 d. 20 m. from the *Weſt*-point thereof towards the *North*, then may I be aſcertain'd that my *Compaſs* hath no *Variation*, but that the *Fly* or *Wires* do point directly *North* and *South*.———But finding before-hand the *Amplitude* to be 33 d. 20 m. and I ſhould find the Sun to riſe but 28 degr. from the *Eaſt*-point of my Compaſs, then ſubſtracting 28 degr. from 33 degr. 20 min. the difference is 5 degr. 20 min. and ſo much doth my *Compaſs* varie from the true *Eaſt*-point, and conſequently all the other Points of the *Compaſs* as much.

Now to find which way the *Compaſs* varieth, you muſt obſerve whether your *Amplitude*, found by your *Calculation*, be to the *Right* or *Left-hand* of the *Sun's riſing* or *ſetting*. And

if it be on the *Right-hand*, you may conclude the *Variation* to be *Easterly*; but if on the *Left-hand*, *Westerly*.

As for *Example*; Finding by the *Amplitude* that the Sun should rise 33 d. 20 min. from the *East Northerly*, when I come to set my *Compass* to the Sun at his *rising*, I find that the Sun riseth but 28 degr. from the *East Northerly*; wherefore the *Amplitude* found is on the *Left-hand*, and so I conclude the *Variation* to be 5 d. 20 min. *Westerly*.

II. *To find the Variation by the* Azimuth.

SUppose the Sun's *Azimuth* found by the Eleventh *Proposition* to be 107 degr. 30 min. from the *North*, and when I set the *Compass*, I find the *Magneticall Azimuth* to be 102, the difference between the *true* and the *Magneticall Azimuth* being 5 d. 30 m. which is the *Variation*.

Now to know whether this *Variation* be towards the *East* or towards the *West*: seeing by the *Azimuth* found the Sun should have been 107 d. 30 min. from the *North*, which is 17 degr. 30 min. from the *East*; but setting of the *Sun* with my *Compass*, I find that it was from the *East* to the *Southward* onely 12 degr. so that the Degree upon which the Sun should have been was more towards the *Right-hand* then the Degree on which it was; therefore I conclude the *Variation* to be 5 degr. 30 min. *Easterly*.

PRO-

PROPOSITIONS
GEOGRAPHICALL,

Shewing how the Diſtance of any two Places upon the Terreſtriall Globe may be found, both by Trigonometricall Calculation and Geometricall Projection.

The Eighth EXERCISE.

BEfore I come to ſhew how the Diſtances *of* Places *are to be found upon the* Terreſtriall Globe, *it will be neceſſary, firſt, to deſcribe unto you the manner how any two* Places, *whoſe* Longitudes *and* Latitudes *are given, may be laid down upon the* Projection. *Wherefore in the* Scheme, *the outward Circle thereof, N Æ S æ, repreſents the* firſt Meridian, *paſſing over the* Iſlands *of* Azores, *from whence the* Ancients *did begin their account of* Longitude, *becauſe (ſay ſome) the* Compaſs *hath there no* Variation. *——The Line Æ æ is the* Æquinoctial, *upon which the* Longitude *is counted from the* firſt Meridian. *—The Circles N E S, N R S, N C S, and N D S, are* Circles of Longitude *paſſing over ſeverall Places. ——The leſſer* Circles *E, C, D, and V, are Circles or* Parallels of Latitude.

*—The Points *, *, *, &c. are ſeverall Places whoſe diſtance we are to find by the following* Propoſitions.*—And the Points ⊙, ⊙, ⊙, &c. are the Poles of the Arches of great Circles which paſs through the reſpective Places whoſe diſtance is to be found.*

And here note, *That the* Circles of Longitude *in this* Projection *are the ſame as the* Azimuth Circles *in the former* Projection; *and the* Centres *and* Poles *of them are found in the ſame manner.——Likewiſe, the* Parallels of Latitude *in this* Scheme *are the ſame with the* Parallels *or* Circles of Altitude *in the former* Projection, *and their* Centres *are found in the ſame manner as is before, in the Deſcription of that* Projection, *preſcribed; and therefore it ſhall here need no more Precepts for its Delineation, but we will proceed to the* Propoſitions *which ſhew how to find the* Diſtance of Places.

Prop. I.

Two Places which differ onely in Latitude, *to find their* Diſtance.

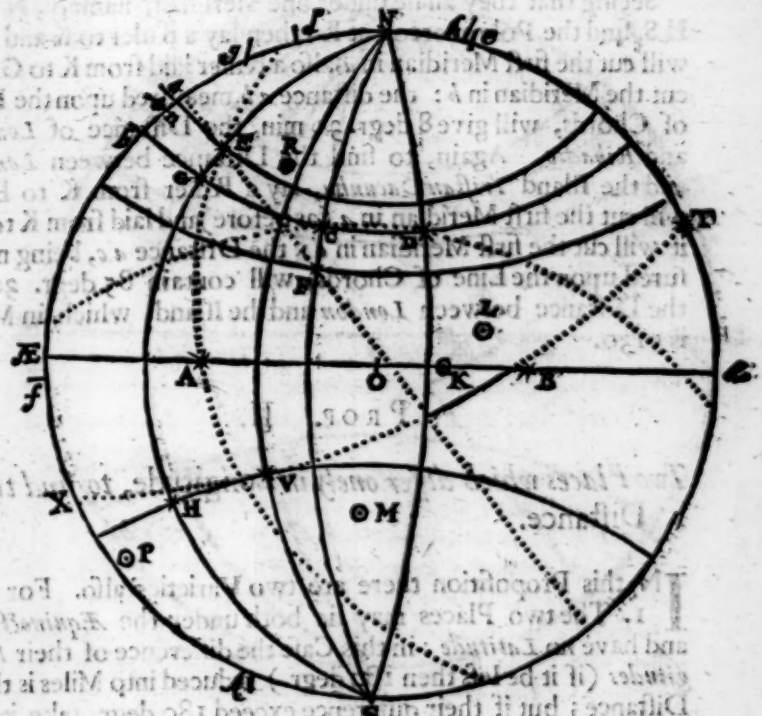
IN this Propoſition there are two Varieties.

1. If both the *Places* lie under one and the ſame *Meridian*, and on one and the ſame Side of the *Æquinoctial*, either on the *North* or *South* Side thereof, then *ſubſtract* the *leſſer Latitude* from the *greater*, and the Difference converted into *Miles* (by allowing 60 *Miles* to *one Degree*) ſhall give you the Diſtance.

Example. *London* and *Ribadio* lie both under one *Meridian*, namely of 20 degr. of Longitude; but they differ in Latitude, for *London* hath 51 d. 30 min. and *Ribadio* hath Latitude 43 d. both *North*; the difference of Latitude is 8 degr. 30 m. which being turned into Miles makes 510 miles.

2. If the two Places lie under one and the ſame *Meridian*, but

but one on the *North*, and the other on the *South*-ſide of the *Æquinoctial*, adde both the Latitudes together, the Sum is the Diſtance.

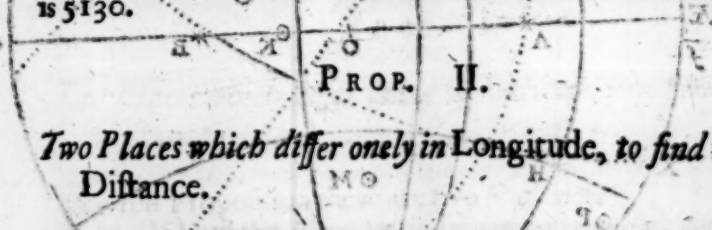


Example. *London* and the Iſland *Triſtan Dacunha* lie both under one *Meridian*; but *London* hath 51 degr. 30 min. *North* Latitude, and the Iſland hath 34 d. *South* Latitude: their Sum is 85 degr. 30 min. which converted into Miles (by dividing

the Degrees by 60. and allowing for every Minute one Mile) makes 5130 miles. And such is the distance of *London* and the Island *Tristan Dacunha*.

To find the distance of these Places upon the Projection.

Seeing that they all lie under one Meridian, namely, N E G H S, find the Pole thereof at K; then lay a Ruler to K and E, it will cut the first Meridian in *a*; also a Ruler laid from K to G will cut the Meridian in *b*: the distance *a b*, measured upon the Line of Chords, will give 8 degr. 30 min. the Distance of *London* and *Ribadio*. Again, to find the Distance between *London* and the Island *Tristan Dacunha*, lay a Ruler from K to E, it will cut the first Meridian in *a*, (as before) and laid from K to H, it will cut the first Meridian in *c*: the Distance *a c*, being measured upon the Line of Chords, will contain 85 degr. 30 m. the Distance between *London* and the Island, which in Miles is 5130.



Prop. II.

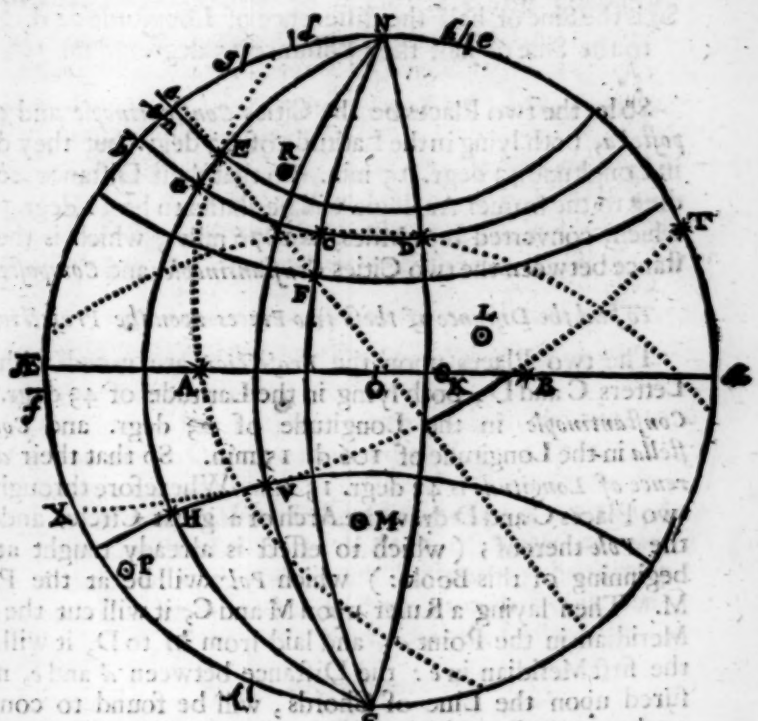
Two Places which differ onely in Longitude, *to find their* Distance.

IN this Proposition there are two Varieties also. For 1. The two Places may lie both under the *Æquinoctial*, and have no Latitude: in this Case the difference of their *Longitudes* (if it be less then 180 degr.) reduced into Miles is their Distance; but if their difference exceed 180 degr. take it out of 360 degr. the remaining Degrees turned into Miles will be the Distance of the two Places.

Example. The Island *Sumatra* and the Island of *S. Thoma* lie both under the *Æquinoctial*, the Island of *S. Thoma* having 33 d. 10 m. of Longitude, and the Island *Sumatra* 137 d. 10 m. The lesser Longitude taken from the greater leaves 104 d. 0 m.

which

which converted into Miles is 6240. And that is the Diſtance of the two Iſlands.



2. But if the two Places differ onely in *Longitude*, and lie not under the *Æquinoctial*, but under ſome other intermediate *Parallel of Latitude*, between the *Æquinoctial* and one of the *Poles*, then to find their Diſtance, this is

The

The Analogie or Proportion.

As the Radius 90 degr. is to the Co-ſine of the common Latitude 47 degr.
So is the Sine of half the difference of Longitude 21 d. 37 m. to the Sine of half their Diſtance 15 degr. 38 m.

So let the two Places be the Cities *Conſtantinople* and *Compoſtella*, both lying in the Latitude of 43 degr. but they differ in Longitude 43 degr. 15 min. So that their Diſtance according to the former Analogie will be found to be 31 degr. 16 m. which, converted into Miles, is 1876 miles, which is the Diſtance between the two Cities *Conſtantinople* and *Compoſtella*.

To find the Diſtance of theſe two Places upon the Projection.

The two Places upon the *Projection* are noted with the Letters C and D, both lying in the Latitude of 43 degr. but *Conſtantinople* in the Longitude of 63 degr. and *Compoſtella* in the Longitude of 106 d. 15 min. So that their *difference of Longitude* is 43 degr. 15 min. Wherefore through the two Places C and D draw the Arch of a great Circle, and find the *Pole* thereof; (which to effect is already taught at the beginning of this Book:) which *Pole* will be at the Point M. Then laying a Ruler upon M and C, it will cut the firſt Meridian in the Point *d*; and laid from M to D, it will cut the firſt Meridian in *e*: the Diſtance between *d* and *e*, meaſured upon the Line of Chords, will be found to contain 31 degr. 16 min. which, converted into Miles, giveth 1876, the Diſtance as before.

PROP.

Prop. III.

Two Places differing both in Longitude *and* Latitude *being propoſed, to find their* Diſtance.

THERE are three Varieties contained in this *Propoſition.* For

1. One of the Places may lie under the *Æquinoctial*, and have no *Latitude*; and the other under ſome *Parallel of Latitude*, between the *Æquinoctial* and one of the *Poles*. For finding the Diſtance of Places that are ſo ſituate, this is

The Analogie or Proportion.

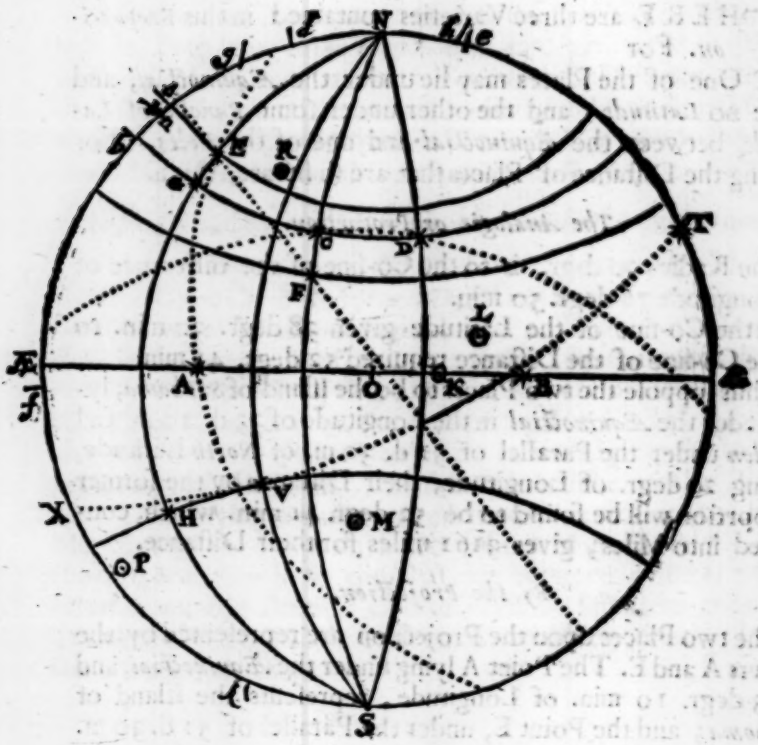
As the Radius 90 degr. is to the Co-ſine of the difference of Longitude 76 degr. 50 min.
So is the Co-ſine of the Latitude given 38 degr. 30 min. to the Co-ſine of the Diſtance required 52 degr. 41 min.

This ſuppoſe the two Places to be the Iſland of *S. Thoma*, lying under the *Æquinoctial* in the Longitude of 33 d. 10 m. and *London* under the Parallel of 51 d. 30 m. of *North* Latitude, having 20 degr. of Longitude, their Diſtance by the former Proportion will be found to be 52 degr. 41 min. which, converted into Miles, gives 3161 miles for their Diſtance.

By the Projection.

The two Places upon the Projection are repreſented by the Letters A and E. The Point A lying under the *Æquinoctial*, and in 33 degr. 10 min. of Longitude, repreſents the Iſland of *S. Thoma*; and the Point E, under the Parallel of 51 d. 30 m. *North*, and in Longitude 20 degr. repreſents *London*.—— Through the two Places A and E (according to former Directions) draw an Arch of a great Circle, and find the *Pole* thereof, which will be at the Point L. A Ruler laid to L and

the Point E will cut the first Meridian in *n* 3 and the Ruler being laid from L to A will cut the first Meridian in *f*: the Distance *n f*, being measured upon the Line of Chords, will be found to contain 52 degr. 41 min. as before, which in Miles is 3161.



2. If both the Places proposed shall be without the *Æquinoctial*, but on one Side, either both towards the *North*, or both

both towards the *South*, the finding of their Diſtance is by this

Analogie or Proportion.

(1.) As the Radius 90 degr. is to the Co-ſine of the difference of Longitude 44 degr.
So is the Tangent of 38 degr. 30 min. to a fourth Tangent 28 degr. 55 min. which taken from the Complement of the Lat. of *Jeruſalem* 58 degr. 20 min. leaves 29 d. 25 m.
(2.) As the Co-ſine of the fourth Tangent 61 degr. 35 min. is to the Co-ſine of 60 degr. 35 min.
So is the Co-ſine of the Latitude of *London* 38 degr. 30 min. to the Co-ſine of the Diſtance 51 degr. 9 min.

So the two Places propounded being *London*, lying in *North* Latitude 51 degr. 30 min. and Longitude 20 degr. and the other, *Jeruſalem*, lying in *North* Latitude alſo 31 degr. 40 min. and Longitude 66 degr. you may find their Diſtance by the foregoing Analogie to be 38 degr. 51 min. which in Miles makes 2331.

By the Projection.

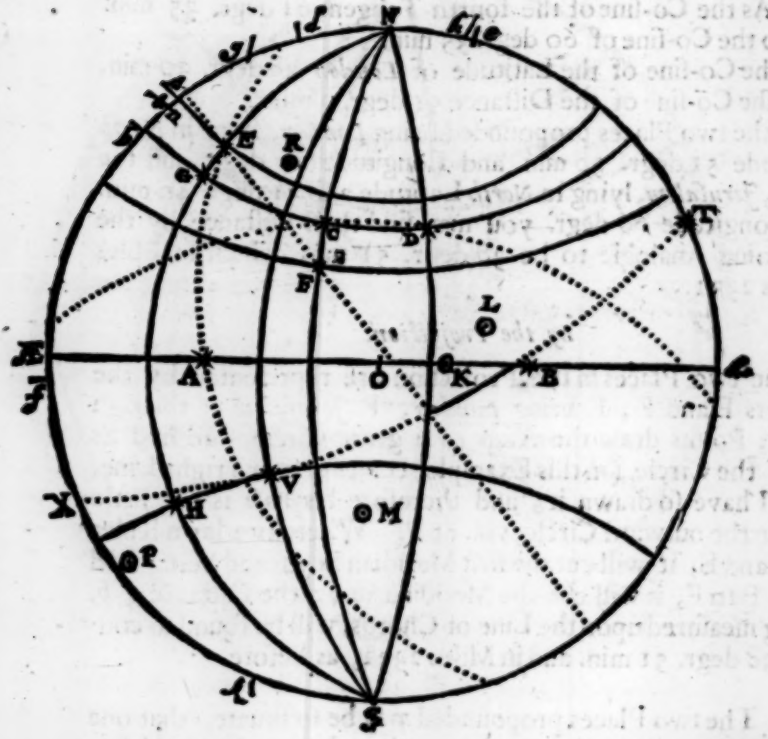
The two Places in the Projection are repreſented by the Letters E and F, E being *London*, F *Jeruſalem*; through which Points draw the Arch of a great Circle, and find its *Pole*: the Circle (in this Example) comes ſo near a right Line, that I have ſo drawn it; and therefore his *Pole* is but little within the outward Circle, viz. at P. Wherefore lay a Ruler to P and E, it will cut the firſt Meridian in *g*; and being laid from P to F, it will cut the Meridian in *h*: the Diſtance *g h*, being meaſured upon the Line of Chords, will be found to contain 38 degr. 51 min. and in Miles 2331, as before.

3. The two Places propounded may be ſo ſituate, that one of them may lie on the *North*, and the other on the *South*-ſide of the *Æquinoctial*. For finding the Diſtance of ſuch Places follow this

Analogie or Proportion.

(1.) As the Radius 90 degr. is to the Co-ſine of the difference of Longitude 40 degr.

So is the Co-tangent of the greater Latitude 50 d. to the Tangent of a fourth Arch 37 d. 10 m. which being ſubſtracted out of the other Latitude, and 90 d. added thereto, ſay,



(2.) As

(2.) As the Co-ſine of the Arch found 52 degr. 50 min. is to the Co-ſine of the Arch remaining 52 degr. 50 min. So is the Co-ſine of the Latitude firſt taken 50 degr. 00 min. to the Co-ſine of the Diſtance 40 degr. which taken from 180 degr. there remains 140 degr. for the Diſtance of the two Places.

So the two Places propounded being the *Cape of Good hope*, lying in the Latitude of 40 degr. *South*, and Longitude 50 d. and the other Place *Malibrigo*, lying in 26 degr. of *North* Latitude, and in 180 degr. of Longitude, you may find their Diſtance by the foregoing Analogie to be 140 degr. which, converted into Miles, make 8400. And ſuch is the Diſtance of the two Places.

By the Projection.

In the Projection the two Places are repreſented by the Letters T and V; the Letter V repreſenting the *Cape of Good hope*, and T *Malibrigo*. Now *Malibrigo* having 180 degr. of Longitude, (which is juſt half the Circumference of the *Æquinoctial*, and is as far remote as any Place can be from the firſt Meridian; for if you were to project any Place having above 180 d. Longitude, (as ſuppoſe 230 degr.) you muſt ſubſtract ſuch Longitude from 360 degr. and project the remainer; ſo 230 degr. being taken from 360 degr. leaves 130 degr. which muſt be projected in ſtead of 230 degr. and by this means it is that *Malibrigo* is projected upon the outermoſt Circle or firſt Meridian.)

Through theſe two Points T and V draw the Arch of a great Circle, T V X, and find its *Pole* at R: then a Ruler laid at R and the Point V will cut the firſt Meridian in *k*, and T *k*, being meaſured upon your Line of Chords, will be found to contain 140 d. and that is their Diſtance, which in Miles maketh 8400.

Theſe are all the Varieties *of* Poſitions *of* Places *upon the* Terreſtriall Globe; *for no two Places (whoſe Diſtance can be*

required) can be proposed, but they must fall under one or other of the Varieties *contained in some of these three* Propositions. *And note that this way of finding* the Distance of Places *is the most* absolute *and* exact *of any other.——And what is here said concerning finding these* Distances *the ingenious may apply to* Circular Sailing, *of all other waies the most perfect: which I shall leave to the* industrious Sea-man *to find out of himself, till I present him with something of that kind: in the mean time let him make use of the foregoing* EXERCISES, *and this which follows.*

THE

The Doctrine of *RIGHT-LINED TRIANGLES* applied to Practice in NAVIGATION:

Whereby

Sundry *Nauticall Queſtions* are reſolved; and many *Problems* of *Sailing*, both by the *Plain* and *Mercator*'s Chart, performed by *Protraction*, by *Calculation*, and alſo wrought upon the *Chart* it ſelf.

The Ninth EXERCISE.

SECTION I.

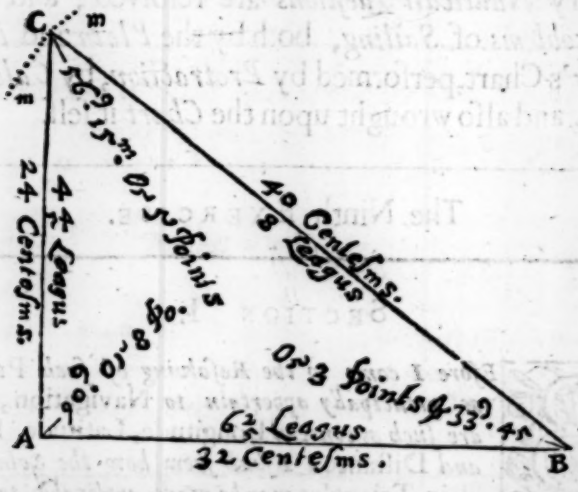
BEfore I come to the Reſolving of ſuch Problems *as principally appertain to* Navigation, *which are ſuch as concern* Longitude, Latitude, Rhumb *and* Diſtance; *I ſhall ſhew how the Solution of* plain Triangles *may be made applicable to the taking of* Heights *and* Diſtances, *and ſo (in the firſt place) propoſe and work ſeverall* Nauticall Queſtions, *which to the induſtrious*

ſtrious Mariner *will be both* delightfull *and* profitable, *and give occaſion to him to invent and put in Practice others of his own contrivance.*

QUESTION I.

There are two Ships ſet ſail from the Port A, the one ſaileth directly North 24 Centeſms, (or 4 Leagues and ⅖ parts of a League,) and the other directly Eaſt 32 Centeſms, (or 6 ⅖ Leagues;) I demand how the two Ships bear one from the other, and alſo how far they are aſunder.

DRAW a right Line A B, and upon A raiſe the Perpendicular A C: let the Point A repreſent the Port from whence the two Ships ſet ſail: then, becauſe the firſt Ship



ſailed 24 Centeſms North, from a Scale of equal parts take 24 Cent. and ſet them from A to C; ſo ſhall C be the place of the firſt Ship. Then, becauſe the other Ship ſailed directly East,

Eaſt, which is a Quadrant or Quarter of the Compaſs diſtant from the North, therefore the Angle at A muſt be a right Angle: And becauſe the ſecond Ship ſailed Eaſt 32 Cent. take 32 Cent. from the ſame Line of equal parts, and ſet them upon the Line A B, from A unto B; ſo ſhall B be the place of the ſecond Ship.

Now firſt, To know how theſe two Ships bear one from another, Draw the Line C B, and meaſure the Quantity of the Angle at B, which you ſhall find to be 33 degr. 45 min. which is three Points from the Weſt Northerly, that is the N. W. by Weſt Point of the Compaſs; and ſo doth the ſecond Ship B bear from the firſt Ship C.——Again, find the quantity of the Angle at C, which you ſhall find to be 56 degr. 15 m. which is five Points from the South Eaſterly, that is the S. E. by Eaſt Point of the Compaſs; and ſo do the two Ships bear one from the other.——Then for the Diſtance that the two Ships are from each other, Take in your Compaſſes the diſtance between B and C, which meaſure upon your Scale of equal parts, and you ſhall find it to contain 40 Centeſms or 8 Leagues; and ſo far aſunder are the two Ships B and C.

The Bearing of the Ships one from the other is found by the firſt Caſe of Right-angled plain Triangles by this Analogie.

As the Diſtance that the firſt Ship ſailed is to the Diſtance that the ſecond Ship ſailed;
So is the Radius to the Tangent of the Angle that the firſt Ship bears to the ſecond. The Complement whereof is the bearing of the ſecond Ship to the firſt.

The Diſtance of the Ships from each other is found by the ſeventh Caſe, by the following Analogie. Having found the Angle C, the bearing of the firſt Ship from the ſecond, ſay,

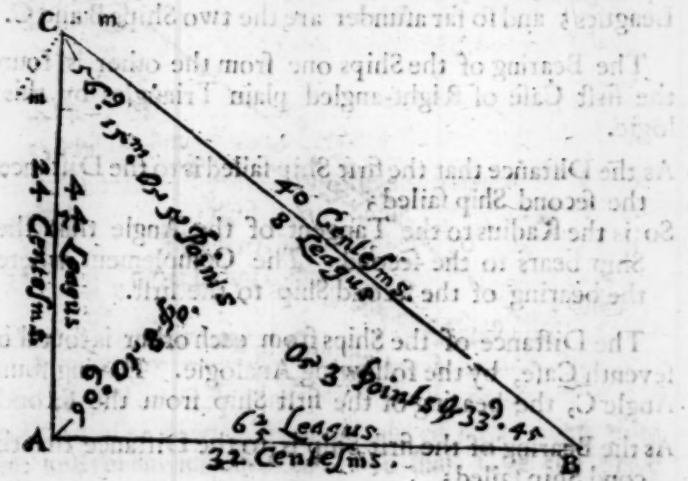
As the Bearing of the firſt Ship is to the Diſtance that the ſecond Ship ſailed;

So is the Radius to the Diſtance of the two Ships.

QUEST. II.

A Ship at A diſcovers an Iſland at C, lying from her directly Eaſt, but ſhe ſails from A towards B 32 Cent. or 6 ½ Leagues directly South; but her Compaſs coming to ſome miſchance, that uſe cannot be made of it, ſhe again at B diſcovers the ſame Iſland, and ſails upon an unknown Point of the Compaſs directly upon the Iſland, and touches upon it, having ſailed 8 Leagues.—I demand upon what Point of the Compaſs the Ship ſailed from B to C, and alſo how far off the Iſland was from A, where it was firſt diſcovered.

DRAW a Line C A, repreſenting a Line of Eaſt and Weſt, and upon A erect a Perpendicular A B, and from A to B ſet off 32 Cent. or 6 ½ Leagues, the diſtance that the Ship ſailed from A to B. Then take out of your Scale of equal



part

parts 40 Cent. or 8 Leagues, the diſtance that the Ship ſailed from B to the Iſland; and ſetting one foot of the Compaſſes in B, with the other deſcribe an obſcure Arch of a Circle *m m*, croſſing the Eaſt and Weſt Line in C: ſo is C the place of the Iſland.

Now firſt, to find upon what Point of the Compaſs the Ship ſailed from B to the Iſland, you muſt find the quantity of the Angle at B, (either by your Line of Chords, or Protracting Quadrant,) and you ſhall find it to contain 33 degr. 45 min. which is three Points from the North Eaſterly, that is N. E. by N. and upon that Point did the Ship ſail from B to the Iſland at C.——Then, to know how far the Iſland C was from A, where it was firſt diſcovered, Take in your Compaſſes the length of the Line A C, and meaſure it upon your Scale; ſo ſhall you find that to contain 24 Cent. or 7 Leagues: and ſo far diſtant was the Iſland from A.

The Point of the Compaſs that the Ship ſailed upon from B to C may be found by the ſecond Caſe of Right-angled plain Triangles, by this Analogie.

As the Diſtance which the Ship ſailed from B to C is to the Radius;

So is the Diſtance ſailed between A and B to the Co-ſine of the Point that the Ship ſailed upon from B to C.

The Diſtance that the Ship was from the Iſland, when firſt diſcovered, may be found by the fifth Caſe of Right-angled plain Triangles, by the following Analogie.

(1.) As the Diſtance that the Ship ſailed from B to C is to the Radius;

So is the Diſtance that the Ship ſailed from A to B to the bearing of the Iſland from B.

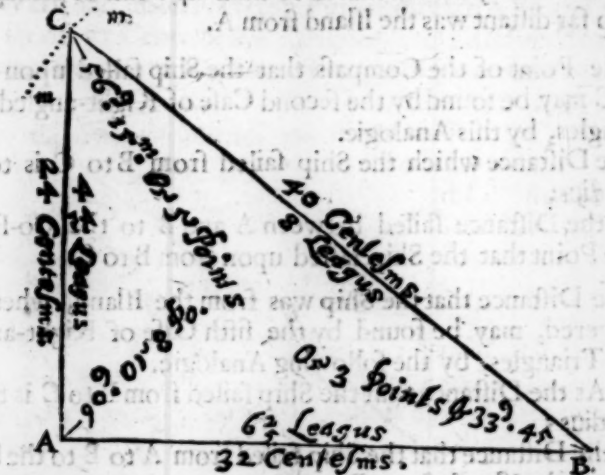
(2.) As the Radius is to the Diſtance that the Ship ſailed from C to B;

So is the Sine of the Rhumb that the Ship ſailed upon from B to C to the Diſtance of the Iſland from A.

Quest. III.

There are two Ports at A and B, which are distant 6 ½ Leagues, and lie directly North and South of each other; from whence two Ships set sail, both for the Port C: the Ship at B sails away upon a South-W. by South Point, and the Ship at A sails directly West.——I demand how many Leagues either of the Ships had sailed when they met at the Port C, and also how the Port C did bear from that at B.

DRAW a right Line A B, and upon it set off 32 Centesms, or 6 ½ Leagues. Now because the Ship at B steered a S. W. by S. Course, which is three Points from the



South-Westerly, therefore upon the Point B protract an Angle of 33 degr. 45 min. and draw the Line B C.——Then, because the Ship at A steered a Westerly Course, which is a Quarter from the North, upon the Point A protract an Angle of

of 90 degr. and draw the Line A C, cutting the former Line B C in C.———Now to know how many Leagues each Ship ſailed, take in your Compaſſes the length of the Line B C, and meaſuring it upon your Scale, you ſhall find it to contain eight Leagues; and ſo many did the Ship that came from B ſail. Alſo take the length of the Line A C in your Compaſſes, and meaſuring that upon your Scale, it will be found to contain 24 Centeſm. or 4 ⅘ Leagues; and ſo much did the Ship that came from A ſail. Now to know how the Port at C did bear from that at B, find the quantity of the Angle at C, which you ſhall find to be 56 degr. 15 min. that is, five Points from the Eaſt Northerly, namely, N. E. by N. and ſo did the Port C bear from B.

The finding of the Diſtance that each Ship ſailed may be done by the third Caſe of Right-angled plain Triangles by this Analogie.

As the Diſtance of the two Ports A and B is to the bearing of the Port C from B;
So is the Sine of the Rhumb that the Ship ſailed upon from B to C to the Diſtance that the Ship ſailed from A to C;
And ſo is the Radius to the number of Leagues that the Ship ſailed from B to C.

QUEST. IV.

A Ship at C diſcovers a Point of Land at A bearing from her S. S. E. but ſhe ſhapes a Courſe E. by S. and ſails away 8 Leagues to B, and at B ſhe diſcovers the ſame Point of Land bearing from her W. S. W.———I demand how far the Ship was from Land being at C and B.

DRAW a Line C B containing 40 Cent. or 8 Leagues, and upon C protract an Angle of 56 degr. 15 min. or five Points, which is the difference the Point of Land did bear

from the Ship being at C, and the Point upon which he ſailed from C to B; and draw a right Line C A.——Then upon the Point B protract an Angle of 33 degr. 45 min. which is the difference of the Ship's bearing from C and A, ſhe being at B, namely, W. S. W. and draw the Line B A, cutting the Line C A, before drawn, in A.

Now to find how far the Ship was from Land being at C, meaſure the Line C A upon your Scale of equal parts, and you ſhall find it to contain 24 Centeſms, or 4½ Leagues: and ſo far was the Ship from the Land, when ſhe was at C. Alſo meaſure the length of the Line B A, and you ſhall find that to contain 32 Cent. or 6½ Leagues: and ſo far from Land was the Ship being at B.

To find theſe Diſtances by the Canon of Sines and Table of Logar. you may doe it by the fourth Caſe of Right-angled Triangles, by this Analogie.

As the Radius is to the Diſtance that the Ship ſailed from C to B;
So is the Bearing of the Ship, being at C, to her Diſtance from Land, being at B.

Or,

The Bearing of the Sip, ſhe being at B, to her Diſtance from Land at C.

QUEST. V.

A Ship being at A diſcovers two other Ships at C and B; the Ship at C bears from her directly Eaſt, and the other Ship at B bears from her directly South. The Ship at A ſails directly South 32 Cent. to B, and being at B, ſteers away upon an unknown Courſe to C 40 Cent. or 8 Leagues.——I demand upon what Point the Ship ſailed from B to C,——and alſo how far C is diſtant from A.

Draw

DRAW a right Line A B, for the bearing of the Ship B from the Ship A, which was direct South. Alſo from A draw another Line A C, for the bearing of the Ship C from the Ship A, which was directly East. Now becauſe between the South and the Eaſt is 90 degr. or one Quarter of the Compaſs, therefore upon the Point A protract an Angle of 90 degr. drawing the Lines A C and A B at right Angles. This done, take 32 Cent. out of your Scale of equal parts, which is the diſtance that the Ship ſailed South from A to B. Then take from the ſame Scale 40 Cent. which is the diſtance that the Ship ſailed from B to C upon an unknown Point. And with this diſtance, ſetting one foot of the Compaſſes in B, with the other deſcribe an obſcure Arch of a Circle *m m*, cutting the Line A C in the Point C, and draw the Line C B.—Now to find upon what Point of the Compaſs the Ship ſailed from B to C, find the quantity of the Angle at B, which you ſhall find to contain 33 degr. 45 min. that is three Points from the North Eaſterly, namely, N. E. by N. and upon that Point did the Ship ſail from B to C.—Then to find how far C is diſtant from A, Take the Line C A in your Compaſſes, and meaſuring it upon your Scale, you ſhall find it to contain 24 Cent. or 4½ Leagues: and ſo far is C diſtant from A.

The Point upon which the Ship ſailed from B to C may be found by the ſecond Caſe of Right-angled Triangles, by this Analogie.

As the Diſtance that the Ship ſailed from B to C is to the Radius;
So is the Diſtance that the Ship ſailed from A to B to the Coſine of the Rhumb from the Meridian.

Then for the Diſtance of C from A.

As the Radius is to the Diſtance that the Ship ſailed from B to C;

So

So is the Rhumb from the Meridian that the Ship ſailed upon from B to C to the Diſtance of C A.

QUEST. VI.

Two Iſlands at A and C are diſcovered by a Ship at B, the Iſland A bears from the Ship at B N. N. W. and the Iſland at C bears N. by E. from B; the Ship being at B ſails away N. N. W. to the Iſland A, and having ſailed 32 Cent. touches upon the Iſland, and being there findes that the Iſland C bears from the Iſland A E. N. E.—I demand how far the Ship at B was from the Iſland C, and alſo how far the two Iſlands were aſunder.

DRAW a Line A B, and upon it ſet 32 Cent. which is the Diſtance that the Ship ſailed from B to the Iſland at A. And becauſe the Iſland A did bear from B N. N. W. and the Iſland at C N. by E. which are three Points, or 33 degr. 45 min. aſunder, upon the Point B protract an Angle of 33 d. 45 min. and draw the Line B C.—Then becauſe the Iſland at C bears from the Iſland at A E. N. E. which is eight Points, or 90 degr. from N. N. W. upon the Point A protract an Angle of 90 degr. and draw the Line A C, cutting the Line B C in C.

Now to find the Diſtance of the Ship being at B from the Iſland C, take the Line C B in your Compaſſes, and applying it to your Scale, you ſhall find it to contain 40 Cent. or 8 Leagues; and ſo far was the Ship at B from the Iſland at C. And to find the Diſtance of the Iſlands one from the other, take C A in your Compaſſes, and meaſure it upon your Scale, you ſhall find it to contain 24 Cent. or 4 ⅘ Leagues; and ſo far diſtant were the Iſlands one from the other.

The Diſtance from A to C may be found by the ſixth Caſe of Right-angled plain Triangles, by this Analogie.

As

As the Co-ſine of the Rhumb that the Ship ſailed upon from B to A is to the Diſtance that the Ship ſailed from B to A;

So is the Radius to the Diſtance of the Ship at B from the Iſland at C.

Then for the Diſtance of the two Iſlands, by the fourth Caſe ſay,

As the Radius is to the Diſtance C B;

So is the Sine of the Difference between the bearing of the two Iſlands from B to the Diſtance of the two Iſlands C and A.

QUEST. VII.

Two Ships ſet out from one and the ſame Port A; the Ship C ſails 24 Cent. or 4 ½ Leagues directly Eaſt, and the Ship B ſails away 32 Cent. or 6 ½ Leagues directly South.——When they have thus ſailed, I demand how far the two Ships are from each other.

DRAW a right Line A B, and ſet off upon it 32 Cent. the Diſtance that the Ship ſailed from A to B South. —Then becauſe the other Ship ſailed directly Eaſt, which is 90 degr. from the South, upon the Point A erect the Perpendicular A C, and upon it ſet off 24 Cent. or 4 ½ Leagues from A to C, which was the Diſtance the other Ship ſailed Eaſt. ——Then draw the Line C B, which being taken in your Compaſſes, and meaſured upon your Scale, will be found to contain 40 Cent. or 8 Leagues. And ſo far are the two Ships from each other.

This Diſtance, by the ſeventh Caſe of Right-angled plain Triangles, may be found by this Analogie.

(1.) As the Diſtance that the Ship ſailed from A to B is to the Diſtance that the Ship ſailed from A to C;

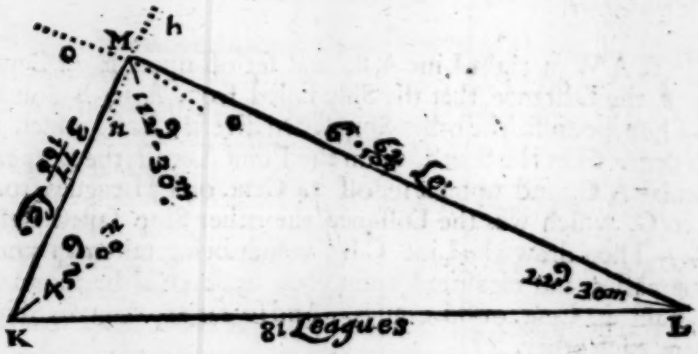
So is the Radius to the Tangent of the Angle at B.

(2.) As the Sine of the Angle at B is to the Diſtance C A; So is the Radius to the Diſtance C B.

QUEST. VIII.

Two Ships ſet ſail from the Port at K; the one ſails 3 $\frac{77}{100}$ Leagues upon the S. W. Point towards M, the other ſails 8 Leagues upon the Weſt Point towards L.——I demand how many Leagues the Ships at M and L are aſunder, and alſo how the Ship at M bears from the Port K, and the other Ship at L.

DRAW a right Line K L, and by help of your Scale ſet off upon it 8 Leagues, the Diſtance that the Ship ſailed from K to L upon the Weſt-Point. Then becauſe the other Ship ſailed 3 $\frac{77}{100}$ Leagues from K towards M upon the



S. W. Point, which is 45 degr. or 4 Points from the Weſt, therefore upon the Point K protract an Angle of 45 degr. and draw the Line K M, ſetting off upon it from K to M 3 $\frac{77}{100}$ Leagues, the Diſtance that the Ship ſailed from K to M, and draw the Line M L.

Now

Now to know, Firſt, how far diſtant the Ships at M and L are from each other, take in your Compaſſes the length of the Line M L, which applie to your Scale, and you ſhall find it to contain 6 $\frac{63}{100}$ Leagues.—And, Secondly, to find how the Ship at M bears from the Port K and the other Ship at L, you muſt find the quantity of the Angle at M, which you will find to be 112 degr. 30 min. that is, eleven Points. Now becauſe the Courſe from K to M was S. W. therefore the Ship at M bears from the Port K N. E. And ſeeing that the Angle at M is 112 degr. 30 min. or eleven Points; therefore eleven Points counted from the N. E. Point is the Bearing of the Ship at M from that at L, which is W. N. W.

The Diſtance of the Ships M and L may be found by the fifth Caſe of Oblique-angled plain Triangles; and the Bearings by the ſecond Caſe.

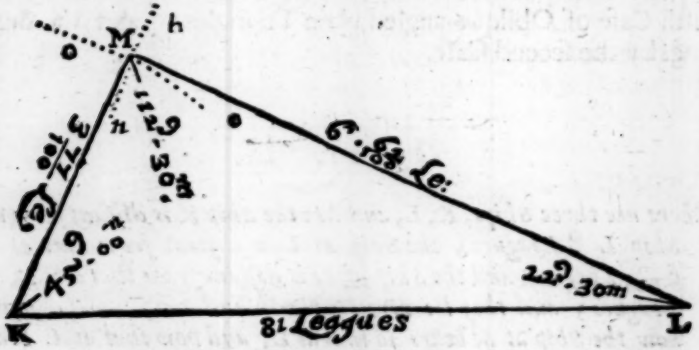
QUEST. IX.

There are three Ships, K, L, and M: the Ship K is diſtant from the Ship L 8 *Leagues; the Ship at L is diſtant from that at M* 6 $\frac{63}{100}$ *Leagues; and the Ship at M is diſtant from that at K* 3 $\frac{77}{100}$ *Leagues; and they lie directly North and South.—I demand how the Ship at M bears to that at L, and how that at L bears to that at K.*

DRAW a right Line, and out of your Scale take 8 Leagues, and ſet them thereon from K to L, for the Diſtance of the Ships at K and L. Then take 3 $\frac{77}{100}$ Leagues, the Diſtance of the Ships K and M, out of your Scale; and ſetting one foot of the Compaſſes in K, with the other deſcribe the obſcure Arch of a Circle *o o*. Again, take 6 $\frac{63}{100}$ Leagues from your Scale, which is the Diſtance that the Ship L was from the Ship M; and ſetting one foot of the Compaſſes in L, with the other deſcribe the obſcure Arch of a Circle *n n*, croſſing

the former Arch in the Point M. Then draw the Lines M K and M L; ſo have you their true Poſitions.

Now to find their Bearing one from another; foraſmuch as the Ships M and K did lie North and South of each other, find the quantity of the Angle at M, which is 112 degr. 30 min. that is, eleven Points from the South Eaſtward, (or 3 Points from the Eaſt Northward,) either of which will be the N.E. by E. Point: and ſo doth the Ship M bear from that at K. And for the Bearing of that at K from that at L, finde the quantity of the Angle at L, which will be 22 degr. 30 min. or two Points; ſo two Points from the S. W. by W. Point Southward is S. W. by S. and ſo doth the Ship L bear to that at K.



The Bearings of the Ships from each other may be found by the third Caſe of Oblique-angled plain Triangles, by the Analogie in that Caſe ſet down.

PRO-

PROBLEMS
Of Sailing by the
Plain Sea-Chart.

SECTION II.

AMONG Sea-men *there are* Three *principal waies of* Sailing *most in* Use *and* Practice: *Two whereof are* Rectilineal, *performed by* Right Lines, *the Third is* Spherical *or* Circular, *performed by* Arches *of* great Circles *of the* Sphere.

Of the Two first, the one is called Plain Sailing, *or*, Sailing *by the* Plain Sea-Chart.

The other is called Mercator's Sailing, *or*, Sailing *by* Mercator's Chart.

These two Charts *are both of them composed of* Right Lines, *yet differ both in their* Construction *and* Use, *though not so much in their* Use *as in their* making *or* composition.

The Plain Sea-Chart *consisteth of* Meridians *and* Parallels, *which are drawn in all parts equal from the* Æquinoctial *towards either of the* Poles; *which is erroneous, as hereafter shall be discovered.*

Mercator's Chart *hath the Degrees of* Longitude *in every* Parallel of Latitude *equal to thoſe in the* Æquinoctial, *as the* Plain Chart *hath: But the Degrees of* Latitude *do increaſe more and more (as they grow nearer the* Poles) *in ſuch a Proportion as every* Parallel of Longitude *doth decreaſe.*

The way of Sailing by the Plain Sea-Chart *is much in uſe, nay too much, conſidering the* Errours *that it leads* Sea-men *into; though they are not ſo eaſily diſcovered in* ſhort *as in* long Voiages, *nor in* places near *the* Æquinoctial *as thoſe* nearer *the* Poles. *But, I ſuppoſe, it is more uſed, for the eaſe there is in* Projecting *of this* Chart, *more then in that of* Mercator's: *otherwiſe I know not why that ſhould be ſo as it is embraced, and the other (I mean that of* Mercator's) *ſo much neglected; which comes ſo near to the* Spherical *way of* Sailing, *that there is an inſenſible difference between them. But I ſhall bring them* face *to* face, *that the* ingenious Sea-man *may ſee their Difference, and thereby abandon* Errour, *and embrace the* Truth. *For in the following* Problems *I ſhall perform the* ſame *thing by* both Charts, *by which the* Errours *may more palpably be diſcovered. And to retain the* Method *which I have obſerved in all the foregoing* EXERCISES, *I ſhall ſhew how theſe* Nauticall Problems *may be* Trigonometrically *performed by the* Tables *of* Logarithms, *and* Canons *of* Artificial Sines *and* Tangents. *And to begin this* Exerciſe, *the firſt thing that I ſhall propoſe unto you is*

The Making of the Plain Sea-Chart.

A *Sea-Chart* may be made either *general*, or *particular*. A *General Sea-Chart* is that whoſe Degrees of *Latitude* proceed from the *Æquinoctial* to either *Pole*, which in the *common Sea-Chart* may be done; but it will be egregiouſly falſe, as the Degrees of *Latitude* grow nearer the *Pole*, as I have already declared.——A *Particular Sea-Chart* is ſuch a one as is made properly for one particular *Navigation*: as if your whole

whole *Navigation* were not to exceed the *Latitudes* of 48 and 60 degr. of *Latitude*, and not to differ in *Longitude* above 8 degrees.

Now to *project* or *make* ſuch a *Chart*; Firſt, draw a right Line A B, repreſenting the *Meridian*, and croſs it at right Angles in the Point A with another right Line A D, repreſenting the *Parallel* of your leaſt *Latitude*, namely, of 48 degr.——— Secondly, conſider what Diſtance you will have your *Parallels of Longitude* and *Latitude* to be, (for in this *Chart* they are both equal,) whether an Inch, 2, 3, or 4 Inches, (for the larger the better.) But in this Example I have made them onely half an Inch. I take therefore half an Inch out of an exact *Scale*, and run it up upon the *Meridian Line* A B, from A to 49, from 49 to 50, from 50 to 51, &c. till I come to my greateſt *Latitude*, which is here ſuppoſed to be 60 degr.——Thirdly, run the ſame Diſtance of half an Inch from A towards D, upon the Line A D, eight times, becauſe the Difference of *Longitude* in your whole *Navigation* will not exceed 8 degrees.——— Fourthly, draw the Line C D, parallel to A B, and B C, parallel to A D, and run the ſame Diſtances upon the Line B C as are upon the Line A D, and the ſame upon C D as are upon the Line A B.———Fifthly, from each Degree of *Latitude* in the Line A B draw to the like Degree of *Latitude* in the Line C D a right Line, as 49,49; 50,50; 51,51; 52,52; &c. till you have drawn all your *Parallels* of *Latitude*.—Sixthly, for your *Meridians*, they are to be drawn in like manner as were the *Parallels* of *Latitude*, all of them equidiſtant, and parallel to your firſt *Meridian* A B, as the Lines 1, 1; 2, 2; 3, 3; &c. And by this means have you the *Meridians* and *Parallels* drawn.

The *grand Diviſions*, or *whole Degrees*, being thus ſet upon your *Chart*, we now come to ſub-divide them. And for the dividing of the Degrees of the *Æquinoctial* at the top and bottom of your *Chart*, let each of them be divided into 5 or 10 parts, and each of thoſe parts ſub-divided into 5 or 10 more

more leſs parts, according as Quantity will permit; for every one of them is ſuppoſed to be divided into 100 or 1000 parts.

For the dividing of the *Degrees of Latitude*; they may be divided as thoſe of *Longitude* were, into 100 parts. But ſometimes each Degree is ſubdivided into 60 *Minutes*, or *Engliſh Miles*, or into 20 *Leagues*.——Now I have divided the *Degrees of Latitude* in this *Chart* each of them into 5 parts, by which means it is capable of the *Numeration* either by *Miles*, *Leagues*, *Centeſms*, or 100 parts.———For if you count by 60 *minutes*, or *miles*, then every of thoſe Diviſions will be 12 *minutes*, or *miles*; if by 20 *Leagues*, then every Diviſion will contain 4 *Leagues*; and if by *Centeſms* or 100 parts, then every of them is 20 *Centeſms*. And thus much concerning the *Making* or *Projecting* of this *Chart*. I now come to ſhew

Some Uſes *of the* Plain Sea-Chart.

THE Problems that are to be reſolved by (or upon) the Sea-Chart are chiefly ſuch as concern *Longitude*, *Latitude*, *Rhumb* or *Courſe*, and *Diſtance*.

Longitude *is the Diſtance of a Place from ſome known* Meridian *to that Place, and is alwaies counted upon the* Æquinoctial.

Latitude *is the Diſtance of any Place from the* Æquinoctial, *counted upon that* Meridian Circle *which paſſeth over that Place.*

Rhumb *or* Courſe *is the* Angle *that a Ship in his Sailing makes with the* Meridian, *and is diſcovered in the general by the* Magneticall Needle, *which alwaies reſpecteth the* North; *and (though not directly, yet) its* Variation *being often obſerved, and the* Chart *rectified thereby, (as I have before ſhewed how it may be done by ſeverall means) is the beſt help that* Navigators *yet have to ſteer their* Courſe *by.*

Diſtance *is the number of* Leagues, Miles, *or* Centeſms, *that any Ship hath* ſailed.

Raiſing

Raiſing of the Pole *is when a Ship ſails from a* leſſer *to a* greater Latitude.

Depreſſing of the Pole *is ſailing from a* greater *to a* leſſer Latitude.

Theſe Terms thus explained, I will proceed to Practice, as followeth.

Probl. I.

How to ſet any Place upon your Chart *according to its Longitude and Latitude.*

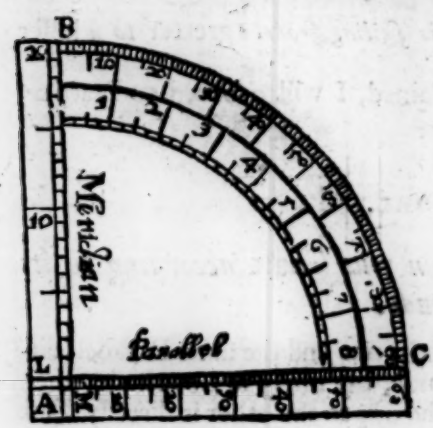
IF the two Places lie under one and the ſame Parallel, differing not at all in Latitude, but onely in Longitude, then the Courſe leading from the one to the other is directly Eaſt or Weſt. As E and F are two Places lying under the Parallel of 50 degr. of Latitude, and differ in Longitude 5 ½ deg. Lay a Ruler to 5 ½ degr. both at the top and bottom of the Chord, and where the Ruler croſſeth the Parallel of 50 degr. as at F, there is your other Place upon the Chart. So E and F lie in 50 degr. of Latitude, and differ in Longitude 5 ½ degr.

But if the two Places to be ſet upon the Chart differ onely in Latitude, and lie under the ſame Meridian as G F, then the Courſe leading from the one to the other is directly North or South, and the Difference of Latitude of F and G is 2 degr. G lying in the Latitude of 48 degr. and F in the Latitude of 50 degr.

But if the Places to be ſet upon the Chart differ both in Longitude and Latitude, as A and F, then the Courſe leading from the one to the other is upon ſome other Point of the Compaſs, ſo far diſtant from the Meridian as is the quantity of the Angle E A F, which here is 70 degr. 1 min. that is upon the E. N. E. Point 2 degr. 31 min. Eaſterly.

This Angle may be found either by Protraction by your Line of Chords, or it may be protracted by your *Protracting*

Quadrant, which in all these Operations upon the Chart is best, for that it avoids the drawing of Arches of Circles upon your Chart or Blank. So then if you were to protract the Angle E A F by your *Protracting Quadrant*; Lay the Centre A of your Quadrant upon the Point A in your Chart, and the Meridian Line of the Quadrant A B upon the Meridian Line of your Chart; then will the Line A C of the Quadrant lie upon the Parallel A D of your Chart: and the Angle that you are to protract being 70 degr. 1 min. by the edge of your Quadrant make a small Mark or Prick with your Needle, and from A through that Point draw a right Line, which will be the Line A F.



And in the same manner as you set any Place upon your Chart, you may find in what Latitudes and difference of Longitude any Places already set upon your Chart are in.

Probl. II.

Any Places being set upon the Chart, to find in what Latitudes they are, and also how they differ in Longitude.

LET the Points Q and R upon the Chart be two Places, and I would know in what Latitudes they lie. First, through the Point Q draw a Line parallel to the Line B C of your Chart; and also through the Point R draw another Line parallel to A C. The Line that is drawn through Q

shoots

ſhoots upon the Latitude of 58 degr. 36 min. and the Line paſſing through R cuts the Meridian of the Chart on either ſide at the Latitude of 57 degr. 16 min. And under thoſe two Latitudes are the two Places Q and R.

Then to find their Difference of Longitude; Take in your Compaſſes the Diſtance between R and S, and meaſuring it upon the bottom of the Chart, it will reach from A to 4 degr. 24 min. And ſuch is the Difference of Longitude of the two Places Q and R.

Probl. III.

Having the Rhumb, and the Diſtance that the Ship hath run upon that Rhumb, to find the Difference of Longitude and Latitude.

The Analogie or Proportion.

As the Radius is to the Diſtance run;
So is the Sine of the Rhumb to the Difference of Longitude:
And
So is the Co-ſine of the Rhumb to the Difference of Latitude.

So the Rhumb being 70 degr. 1 min. that is E. N. E. 2 degr. 31 min. Eaſterly, and the Diſtance run 117 Leagues, the Difference of Longitude will be found to be 5 ½ degr. and the Difference of Latitude 2 degr.

Upon the Chart.

UPON the Point A protract an Angle of 70 d. 1 m. as the Angle E A F, and draw the Line A F, which is the Rhumb upon which the Ship ſailed. Upon this Line ſet 117, the number of Leagues that the Ship ſailed from A to F. Then through the Point F draw the Line F E parallel to A D. So ſhall E F be the Difference of Longitude, 5 d. and an half, and A E the Difference of Latitude, 2 degr.

Probl. IV.

The Difference of Latitude and the Rhumb being given, to find the Diſtance run and the Difference of Longitude.

The Analogie or Proportion.

As the Co-ſine of the Rhumb is to the Difference of Latitude;
So is the Radius to the Diſtance run:
And
So is the Sine of the Rhumb to the Difference of Longitude.

So the one Latitude being 48 degr. and the other 50 degr. the Difference is 2 degr. and the Rhumb being E. N. E. 2 deg. 31 min. Eaſterly, the Diſtance run will be found to be 117 Leagues, and the Difference of Longitude 5 ½ degr.

Upon the Chart.

SET the Difference of Latitude 2 degr. from A to E, and draw the Line E F parallel to A D. Then upon the Point A protract the Angle of the Rhumb 70 degr. 1 min. E. N. E. 2 degr. 31 min. Eaſterly, and draw the Line A F, cutting the other Line E F in F. Then taking in your Compaſſes the length of the Line A F, and meaſuring it upon the ſide of the Chart, you ſhall find it to contain 117; which is the number of Leagues the Ship ſailed: And the Line E F, being ſo meaſured, will contain 5 ½ degr. the Difference of Longitude.

Probl.

Probl. V.

Having the Difference of Longitude and the Rhumb given, to find the Diſtance run and Difference of Latitude.

The Analogie or Proportion.

As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Diſtance run:
And
So is the Co-ſine of the Rhumb to the Difference of Latitude.

So the Rhumb being E. N. E. 2 degr. 31 min. Eaſterly, and the Difference of Longitude 5½ degr. the Diſtance run will be found to be 117 Leagues, and the Difference of Latitude 2 degr.

Upon the Chart.

UPON the Point A protract an Angle of the Rhumb 70 degr. 1 min. and draw the Line A F. Then the Difference of Longitude being 5½ degr. count 5½ degr. upon the bottom of your Chart from A to G, and upon the Point G raiſe a Perpendicular G F, cutting the Line A F before drawn in F. Then the Line A F, being meaſured upon the Side of your Chart, will be found to contain 117 Leagues, the Diſtance run: And F G, there alſo meaſured, will be found to be 2 degr. the Difference of Latitude.

Probl. VI.

The Distance that the Ship hath run, and the Difference of Latitude, given, to find the Rhumb and Difference of Longitude.

The Analogie or Proportion.

As the Distance run is to the Radius;
So is the Difference of Latitude to the Co-sine of the Rhumb:
And
So is the Sine of the Rhumb to the Difference of Longitude.

So the distance run being 117 Leagues, and the Difference of Latitude being 2 degr. the Rhumb will be found to be E. N. E. 2 degr. 31 min. Easterly, and the Difference of Longitude 5 ½ degrees.

Upon the Chart.

SET the Difference of Latitude 2 degr. upon your Chart from A to E, and draw the Line E F parallel to A B. Then out of the Side of your Chart take the Distance run, 117 Leagues; and setting one foot of the Compasses in A, turn the other about till it cross the Line E F, which it will doe in F. Then F E, being measured upon the bottome of your Chart, will contain 5 ½ degr. the Difference of Longitude. And by your Line of Chords or Protracting Quadrant find the Quantity of the Angle E A F, which will be 70 degr. 1 min. the E. N. E. Point 2 degr. 31 min. Easterly.

Probl.

PROBL. VII.

The Distance that the Ship hath run, and the Difference of Longitude, being given, to find the Rhumb and Difference of Latitude.

The Analogie or Proportion.

As the Distance run is to the Radius;
So is the Difference of Longitude to the Rhumb:
And
So is the Co-sine of the Rhumb to the Difference of Latitude.

So the Difference of Longitude being 5 ½ degr. and the Distance that the Ship hath run 117 Leagues; the Rhumb will be found to be E. N. E. 2 degr. 31 min. Easterly, and the Difference of Latitude 2 degr.

Upon the Chart.

COUNT the Difference of Longitude upon the bottome of the Chart from A to G, and upon the Point G raise the Perpendicular G F. Then take out of the Side of your Chart the Distance run, 117 Leagues, and setting one foot of the Compasses in A, with the other cross the Perpendicular F G in the Point F. Now if you take F G in your Compasses, and measure it on the Side of your Chart, you shall find it to contain 2 degr. for the Difference of Latitude; and the Angle E A F, being measured by your Chord or Quadrant, will be 70 degr. 1 min. that is the E. N. E. Point 2 d. 31 m. Easterly for the Rhumb.

PROBL.

Probl. VIII.

The Difference of Longitude and Difference of Latitude being given, to find the Rhumb and the Diſtance run.

The Analogie or Proportion.

As the Difference of Latitude is to the Radius;
So is the Difference of Longitude to the Tangent of the Rhumb:

And

As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Diſtance run.

So the Difference of Longitude being 5 ½ degr. and the Difference of Latitude 2 degr. the Rhumb will be found to be E. N. E. 2 degr. 31 min. Eaſterly, and the Diſtance upon the Rhumb 117 Leagues.

Upon the Chart.

COUNT the Difference of Latitude from A to E, and draw the Line E F parallel to A D. Alſo count the Difference of Longitude from A to G, and upon the Point G raiſe the Perpendicular G F, cutting the Line E F in the Point F. Then take in your Compaſſes the length of the Line A F, and meaſuring it upon the Side of the Chart, you ſhall find it to contain 117 Leagues, the Diſtance that the Ship hath run. And if by your Line of Chords, or Quadrant, you find the Quantity of the Angle E A F, it will be the Rhumb, which you may find to be E. N. E. 2 degr. 31 min. Eaſterly, or 70 degr. 1 min.

Probl.

Probl. IX.

The Rhumb that a Ship hath ſailed upon, and the number of Leagues ſhe hath ſailed upon that Rhumb, being given, to know how much ſhe hath raiſed or depreſſed the Pole.

The Analogie or Proportion.

As the Radius is to the Diſtance run;
So is the Co-ſine of the Rhumb from the Meridian to the Difference of both Latitudes.

So the Rhumb being E. N. E. 2 degr. 31 min. Eaſterly, that is, 70 degr. 1 min. and the Diſtance that the Ship hath ſailed upon that Rhumb 117 Leagues, the Pole will be found to be raiſed 2 degr.

Upon the Chart.

UPON the Point A, the leſſer Latitude, protract an Angle of 70 degr. 1 min. and draw the Line of the Rhumb A F, and out of the Side of your Chart take 117 Leagues, (the Diſtance the Ship ſailed) and ſet them upon the Rhumb from A to F. Then through the Point F draw the Line E F parallel to A D, cutting the Meridian of your Chart in E, which is 2 degr. from A: ſo that the Ship hath raiſed the Pole 2 degrees.

Probl. X.

The Longitude and Latitude of the Place from whence you came, with the Rhumb and Distance sailed, being given, to find the Longitude and Latitude of the Place to which you are come.

The Analogie or Proportion.

As the Radius is to the Distance run;
So is the Sine of the Rhumb from the Meridian to the Difference of Longitude:
And
So is the Co-sine of the Rhumb to the Difference of Latitude.

So the Latitude of the Place from whence you came being 52 degr. and the Longitude 35 degr. the Rhumb upon which you have sailed N.E. by N. 33 degr. 45 min. and the Distance which you have sailed upon that Rhumb 96 $\frac{1}{10}$ Leagues; you shall find the Difference of Longitude to be 2 degr. 40 min. and the Difference of Latitude 4 degr. So that the Place to which you are come is in the Latitude of 56 degr. and in the Longitude of 37 degr. 40 min.

Upon the Chart.

THE Place from whence you came being in the Latitude of 52 degr. and in the Longitude of 35 degr. is represented by H. The Rhumb you have sailed upon being N. E. by N. 33 degr. 45 min. upon the Point H protract an Angle of 33 degr. 45 min. and draw the Line H K for the Rhumb. Then out of the Side of your Chart take 96 $\frac{1}{10}$ Leagues, which is so much as the Ship sailed, and set that upon the Rhumb-Line from H to K, and through the Point K draw the Line K L parallel

parallel to B C, (or perpendicular to A B,) and it will cut the Line A B in L. So K L, being measured on the bottom of your Chart, will be found to contain 2 degr. 40 min. the Difference of Longitude; which added to 35 degr. the Longitude you came from, gives 37 degr. 40 min. for the Latitude you are in. Also the Line H L, being measured on the Side of your Chart, will be found to contain 4 degr. And such is the Difference of Latitude, which added to 52 degr. the Latitude from whence you came, gives 56 degr. the Latitude in which you are.

Probl. XI.

The Longitude and Latitude of the Place from whence you came, the Rhumb upon which you sailed, and the Latitude of the Place to which you are come, being given, to find the Distance and Difference of Longitude.

The Analogie or Proportion.

As the Difference of Latitude is to the Radius;
So is the Tangent of the Rhumb from the Meridian to the Difference of Longitude:
And
As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Distance run.

So the Latitude of the Place from whence you came being 52 degr. and the Longitude 35 degr. and the Rhumb upon which you sailed the third from the Meridian N. E. by N. 33 degr. 45 min. you shall find the Distance run to be 96 Leagues $\frac{1}{10}$, and the Difference of Longitude 2 degr. 40 min.

Upon the Chart.

UPON your Chart assign H for your Place from whence you came, in the Latitude of 52 degr. and Longitude 35 degr. Upon this Point H protract the Angle of the Rhumb 33 d. 45 m. N.E. by N. and draw the Rhumb-Line H K. Then the Latitude of the Place where you are being found by observation (or being otherwise given) to be 56 degr. draw a Line quite cross your Chart at the 56th degree of Latitude, as the Line 56. 56 in the Chart crossing the Rhumb-Line in the Point K. So K L, being measured at the bottome of your Chart, will be found to contain 2 degr. 45 min. which added to 35 degr. the Longitude you came from, makes 37 degr. 40 min. And that is the Longitude in which you are. In like manner measure H K upon the Side of your Chart, and you shall find it to contain 96 7/8 Leagues. And so much hath the Ship run upon that Point N. E. by N.

Probl. XII.

The Latitude of two Places, and the Difference of Longitude between them, being known, to find what Rhumb leadeth from one to the other, and how many Leagues distant they are asunder.

The Analogie or Proportion.

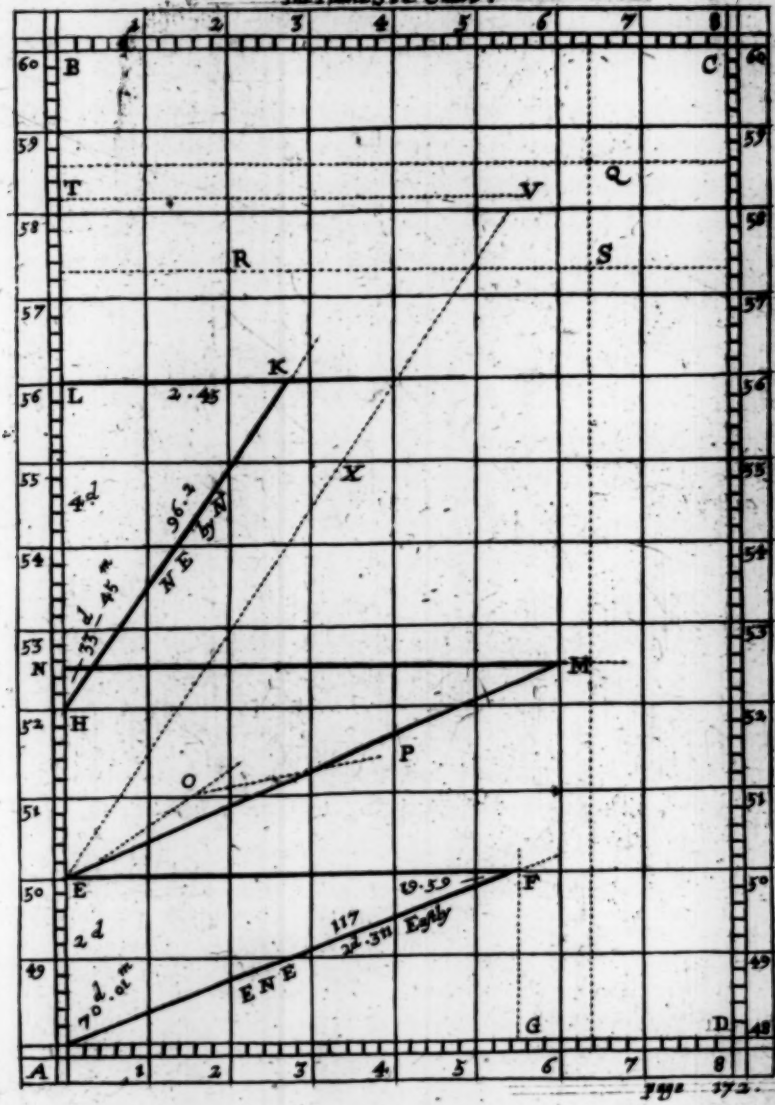
As the Difference of Latitude is to the Radius;
So is the Difference of Longitude to the Tangent of the Rhumb:

And

As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Distance of the two Places.

So

The Plaine Sea Chart.



So the Latitude of one of the Places being 50 degr. and the other 52 degr. 30 min. and the Difference of Longitude 6 ½ degrees; the Rhumb will be found to be 67 degr. 23 min. and the Distance upon the Rhumb 6 ½ degr. or 120 Leagues.

Upon the Chart.

UPON the Point of the greater Latitude at N 52 deg. 30 min. draw a Line N M, parallel to A D, upon which Line ſet 6 degr. the Difference of Longitude of the two Places (being taken from the bottom of the Chart) from N to M. Then from the Point M draw the Line to E, the leſſer Latitude, 52 degr. which Line, taken in the Compaſſes and meaſured upon the Side of the Chart, will be found to contain 6 ½ degr. or 130 Leagues. Alſo the Angle N E M, being meaſured by your Chord, or Protracting Quadrant, will be found to contain 67 degr. 23 min. which is the Rhumb leading from one to the other, namely, ſhort of the E. N. E. Point 7 degr. or, N. E. by E. 21 degr. 8 min. Eaſterly.

PROBLEMS
Of Sailing by
Mercator's Chart.

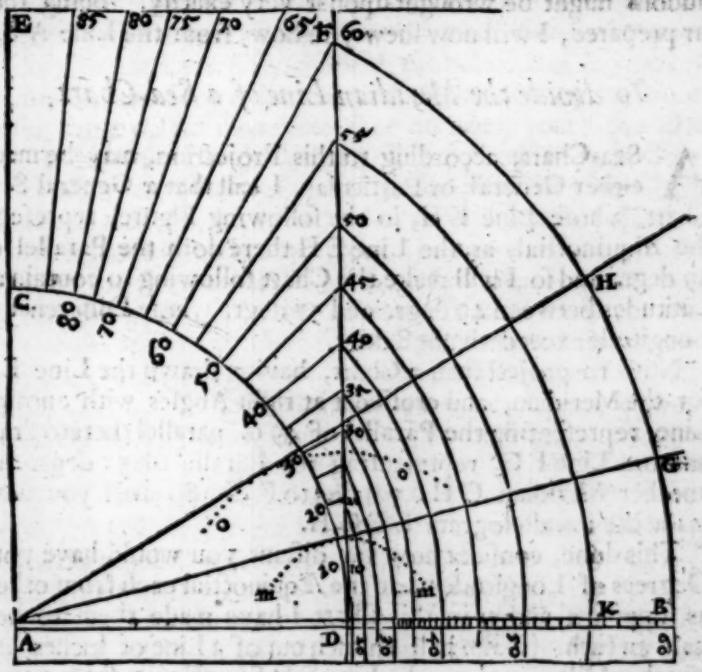
SECTION III.

PROBL. I.

How to make a Sea-Chart according to MERCATOR's Projection, by your Line of Chords.

UPON a piece of thick and ſmooth Paper (or rather Paſt-board) draw a right Line, as A B, and upon the Point A, with 60 degr. of your Line of Chords, deſcribe the Quadrant A C D, which divide into 90 equal Parts or Degrees, as here in this Figure there is onely every fifth Degree. ——This done, upon the Point D erect the Perpendicular D F, (in which you muſt be very exact.) And from the Point A (through each Degree of the Quadrant) draw right Lines, as A 10, 10; A 20, 20; A 30, 30; A 40, 40; &c. towards 90. till they touch the Line D F.——Then with your Com-

Compaſſes, one foot being placed in A, extend the other to 60 degr. in the Line D F, and with that Diſtance deſcribe the Arch F H G B. Again, extend the Compaſſes from A to 55 in the Line F D, and keeping one foot in A, with the other deſcribe 55 K; ſo ſhall the Point K be the Point of 55 degr. in the Line A B.—Alſo, extend the Compaſſes from A to 50 in the Line F D, and draw the Arch 50, 50. Doe ſo with



45, 40, 35, &c. till you come to the beginning of the Degrees at D. So ſhall theſe Arch-lines, by meeting with the Line A B, divide that part of it D B into unequal parts, at 10, 20, 30, 40, 50, 60, and ſo forward. But this Figure is ſufficient for Example.

Now

Now from this Line A B, being thus unequally divided, you may divide the Meridian-line of a Sea-Chart according to *Mercator*'s Projection of any bigneſs, ſo that the Diſtance between Degree and Degree in the Æquinoctial be leſs then the Diſtance A D, which is here two Inches. And if a Chart were made that the Æquinoctial Degrees were two Inches diſtant, and it paſſed upon a ſmooth Board, many Nautical Concluſions might be wrought upon it very exactly. Being thus far prepared, I will now ſhew you how, from the Line A B,

To divide the Meridian Line of a Sea-Chart.

A Sea-Chart, according to this Projection, may be made either General, or Particular. I call that a General Sea-Chart, whoſe Line E H, in the following Figure, repreſents the Æquinoctial, as the Line E H there doth the Parallel of 49 degr. and ſo I will make the Chart following to contain all Latitudes between 49 degr. and 57 degr. whoſe Difference of Longitude exceedeth not 8 degr.

Now to project ſuch a Chart, having drawn the Line E F for the Meridian, and croſſed it at right Angles with another Line repreſenting the Parallel of 49 d. parallel thereto draw another Line F G, repreſenting the Parallel of 57 degr. and another Meridian G H, parallel to F E. So ſhall you have made the Parallelogram E F G H.

This done, conſider how far diſtant you would have your Degrees of Longitude upon the Æquinoctial each from other, as ſuppoſe (and as in this Chart I have made them to be) half an Inch. Take half an Inch out of a Line of Inches, and run that Diſtance along the Line E H from E to 1, from 1 to 2, from 2 to 3, &c. And alſo doe the like upon the Line F G, at the top of the Chart, drawing the Lines 1, 1; 2, 2; 3, 3; &c.

Now for the Dividing of the Meridians E F and H G, repair to the foregoing Figure, taking in your Compaſſes the Diſtance that is between Degree and Degree of the Æquinoctial,

ctial, which in our Example is half an Inch. With this Diſtance, ſet one foot of the Compaſſes in the Point D, and with the other deſcribe the Arch *m m*; by the very Edge whereof draw the Line A G: ſo is your Figure prepared to divide the Meridian-line of a Sea-Chart whoſe Degrees of Longitude are half an Inch diſtant.

Now in reſpect that your firſt Parallel of Latitude E H in your Chart is drawn for 49 degr. your next Parallel muſt be 50 degr. Wherefore ſet one foot of your Compaſſes upon 50 degr. in the Line A B, and with the other take the neareſt Diſtance to the Line A G: that is done by turning the Compaſſes about till the moveable foot do onely touch the Line A G; which when it ſo doth, that Diſtance at which your Compaſſes then are, being ſet upon the Meridian of your Chart, will reach from 49 degr. to 50, which being ſet upon your Chart, on both ſides thereof, from 49 draw the Line 50. 50 will give you the Parallel of 50 d. of Latitude. In like manner for the Parallel of 51 degr. Set one foot of the Compaſſes in 51 degr. upon the Line A B of the former Figure, and with the other take the leaſt Diſtance to the Line A G: this Diſtance ſet upon the Meridian of your Sea-Chart, on both ſides thereof, will reach from 50 to 51; and there draw the Parallel 51, 51.—— Likewiſe for the Parallel of 52 degr. Set one foot of the Compaſſes in 52 degr. in the Line A B, taking the neareſt Diſtance to the Line A G: that Diſtance ſet upon the Meridian of your Sea-Chart, on both ſides thereof, will reach from 51 to 52; and there draw the Parallel of 52, 52. Doe thus with all the Degrees, as 53, 54, 55, 56, and 57. So ſhall the Meridians of your Chart E F and H G be divided into whole Degrees.

For the Sub-diviſions of theſe Degrees, they may be divided each of them into equal parts, as the Diviſions at the top and bottome of the Chart ought to be; but the Degrees of the Meridian, as they grow higher, they ought ſtill to grow greater. But the Difference is ſo ſmall, that it cannot produce

any considerable Errour, though the Sub-divisions be all made equal between Degree and Degree. You may therefore divide them either into 60 Minutes or English Miles, or into 20 Leagues, or into 100 parts of Degrees, as you shall best like of.

But if you would make a Chart that the Distance between Degree and Degree upon the Æquinoctial should be an Inch, or any other Distance less then A D in the foregoing Figure; take that Distance (as suppose an Inch) in your Compasses, and setting one foot in D, with the other describe the Arch o o, and draw the Line A H onely to touch the Arch o o. The least Distance taken from each Degree to this Line A H shall give you the Distance of the Degrees upon the Meridian of a Sea-Chart, whose Distance of Degrees upon the Æquinoctial are an Inch from each other.

Your Chart being thus prepared, I will now come to shew you how to resolve severall Problems upon it.

Probl. II.

To find how many Leagues do answer to one Degree of Longitude in every severall Latitude.

UPON the two edges of your Protracting Quadrant there are two Lines, the one divided into 20, the other into 60 equal parts.

Take therefore the least Distance from the Complement of the Parallel's distance from the Æquator, (or the Complement of the given Latitude:) this Distance, being measured upon the edge that is divided into 20, shall shew you what number of Leagues make one Degree of Longitude in that Parallel of Latitude. And the same Distance, being measured upon the other edge that is divided into 60, will give so many of our Miles, or so many Minutes of the Æquinoctial, or any other great

great Circle, as are anſwerable to one Degree of Longitude in that Latitude.

Example. Let it be required to find how many Leagues do anſwer to one Degree of Longitude in the Latitude of 18 d. 12 min.

Set one foot of your Compaſſes in 71 degr. 48 min. the Complement of the given Latitude, and with the other take the neareſt Diſtance to the ſide of the Quadrant which is divided into 20: that Diſtance, meaſured upon the Line 20, will reach from the beginning thereof to 19: and ſo many Leagues do anſwer to one Degree of Longitude in the Latitude of 18 degr. 12 min.

Or, If you take the leaſt Diſtance from 18 degr. 12 m. the Latitude it ſelf, in the Limb of the Quadrant, to that edge which is divided into 60, that Diſtance will alſo reach to 19 upon the Line 20, as before.

And the ſame Diſtance, being meaſured upon the Line 60 of the Quadrant, will give you 57 parts: and ſo many Minutes of the Æquator are anſwerable to one Degree of Longitude in the Parallel of 18 degr. 12 min. of Latitude.

So likewiſe in the Latitude of 25 degr. 15 min. if you take the leaſt Diſtance from the Complement thereof, or from the Latitude it ſelf, to the edges of the Quadrant, you ſhall find that Diſtance to reach 18 in the Line of 20: and ſo many Leagues do anſwer to one Degree of Longitude in the Latitude of 25 degr. 15 min. or unto 54 in the Line of 60: and ſo many Minutes of the Æquator do anſwer to one Degree of Longitude in that Parallel of Latitude.

The Analogie or Proportion.

As the Radius is to the Co-ſine of the Latitude;
So is { 20 Leagues / 60 Minutes } to the number of { Leagues / Minutes } which anſwer to one Degree of Longitude in that Latitude.

Probl. III.

By the Latitude of two Places and their Distance, to find the Rhumb.

The Analogie or Proportion.

As the Distance upon the Rhumb is to the Radius;
So is the Difference of Latitudes to the Co-sine of the Rhumb from the Meridian.

Thus if the Places given were one in the Latitude of 50 d. and the other in the Latitude of 55 degr. and the Distance upon the Rhumb 6 degr. or 120 Leagues; the Rhumb leading from one to the other will be found to be the third from the Meridian, namely, N. E. by N. 33 degr. 45 min.

Upon the Chart.

LET A represent the Place in the Latitude of 50 degr. and C that in 55 degr. whose Distance from A to C is 6 degr. Take 6 degr. out of the Meridian-line, by setting one foot as much below the lesser Latitude as above the greater, which will be from K in the Latitude of 49 ½ degr. to L in the Latitude of 55 ½ either of which are half a Degree above and under the two given Latitudes. Take this Distance K L in your Compasses, and setting one foot in A, (the lesser Latitude) with the other cross the Parallel of the greater Latitude 55 degr. in the Point C, and draw a right Line from A to C. So shall the quantity of the Angle B A C, being found (either by your Chord or Quadrant,) shew you the Inclination of the Rhumb to the Meridian to be 33 degr. 45 min. the N. E. by N. Point.

Note, *That in the Propositions following, the Difference of Longitude must always be taken out of the Æquator, and measured there-*

thereupon alſo. But the Difference of Longitude and Diſtance upon the Rhumb muſt alwaies be meaſured upon, and taken out of, the Meridian Line of your Chart. And hereafter I ſhall call them the proper Difference, and proper Diſtance.

Probl. IV.

The Longitude and Latitude of two Places being given, to find the Rhumb.

The Analogie or Proportion.

As the proper Difference of Latitude is to the Radius;
So is the Difference of Longitude to the Tangent of the Rhumb from the Meridian.

Thus if the Places ſhould lie one in the Latitude of 50 degr. and the other in the Latitude of 55 degr. and the Difference of Longitude between them were 5 degr. 30 min. the Rhumb leading from one Place to the other will be found to be the third from the Meridian N. E. by N. 33 degr. 45 min.

Upon the Chart.

THE Meridians and Parallels being drawn through the two Places at A and C, and a ſtraight Line from A to C, for the Rhumb, by your Chord or Quadrant find the quantity of the Angle B A C, which you will find to be 33 d. 45 m. or the third Rhumb from the Meridian N. E. by N.

But if this Rhumb were to be found by the Common Sea-Chart, it would be found to be above 47 degr. that is, N. E. 2 degr. Eaſterly, that is, one whole Point and 2 degr. more Eaſterly then it ſhould be.

Probl. V.

The Latitude of two Places and the Rhumb being given, to find the Difference of Longitude.

The Analogie or Proportion.

As the Radius is to the Tangent of the Rhumb from the Meridian;
So is the proper Difference of Latitudes to the Difference of Longitude.

Thus the Latitude of one Place being 50 degr. and the other 55 degr. and the Rhumb leading from one to the other being the third from the Meridian, the Difference of Longitude will be found to be $5\frac{1}{2}$ degr.

Upon the Chart.

LET a Meridian be drawn through A, and a Parallel of Latitude through C. Then upon the Angle A protract the Angle of the Rhumb 33 degr. 45 min. So the Distance B C upon the Parallel, being measured upon the bottome of the Chart, will be found to contain 6 degr. 30 min.

But if this Difference of Longitude were to be found by the Plain Sea-Chart, the Difference of Longitude would be found to be but 3 degr. 20 min. which is more then 3 degr. less then the truth; a vast Difference. And yet this Errour would be yet greater, if either the Latitude be greater, or the Rhumb farther from the Meridian.

Probl. VI.

The Difference of Longitude of two Places, the Latitude of one of them, and the Rhumb leading from one to the other, given, to find the Latitude of the other Place.

The Analogie or Proportion.

As the Radius is to the Co-tangent of the Rhumb from the Meridian;
So is the Difference of Longitude to the proper Difference of Latitude.

Thus if the Latitude of one of the Places were 50 degr. the Rhumb leading from that to the other N. E. by N. 33 d. 45 min. and the Difference of Longitude between the two Places were 5 degr. 30 min. the Latitude of the other Place will be found to be in 55 degr.

Upon the Chart.

LET AB and DG be two Meridians drawn through A and C, at 5 ½ d. the Difference of Longitude, and a Parallel of Latitude through A, crossing the Meridian CD in D. Then upon the Point A protract an Angle equal to the Rhumb from the Meridian given 33 degr. 45 min. So the Line CD, being measured upon the Meridian from A, the given Latitude, 50 degr. will reach to 56 degr. the proper Difference of Latitude. So that the other Place lies in the Latitude of 56 degr.

But if this Difference of Latitude were to be found by the Plain Sea-Chart, this Difference of Latitude would be found to be 8 *d.* 13 *min. and the Latitude sought would be found to be* 58 *degr.* 13 *min.*

13 min. above three Degrees more then the truth. As by the Triangle for that purpose drawn upon the Plain Sea-Chart, marked with T V E, may appear.

Probl. VII.

Having the Latitude of one Place, the Rhumb leading from that Place to another unknown, and the Distance upon the Rhumb from the first to the second Place, to find the Difference of Longitude of the two Places.

The Analogie or Proportion.

As the Radius is to the Sine of the Rhumb from the Meridian; So is the proper Distance upon the Rhumb to the Difference of Longitude.

Thus if the two Places were one in the Latitude of 50 degr. and the other in a greater Latitude, but unknown; the proper Distance upon the Rhumb leading from one place to the other being 6 degr. and the Rhumb N. E. by N. 33 degr. 45 min. the Difference of Longitude will be found to be 5 ½ degr.

Upon the Chart.

THrough the Point A in the Latitude of 50 degr. let be drawn a Meridian A B, and a Parallell A D; and upon the Point A protract an Angle equal to the Rhumb from the Meridian 33 degr. 45 min. Then take with the Compasses 6 degres, the proper Distance upon the Rhumb, out of the Meridian-line, (having respect to the Latitude of the Places) as from K to L, and set that Distance upon the Rhumb from A to C. Then through C draw another Meridian C D, crossing the Parallel drawn through A in the Point D. So the Line A D, being measured at the bottom of the Chart, will be found to contain 5 ½ d. the Difference of Longitude sought.

But

But if this Difference of Longitude had been to be found by the Common Sea-Chart, it would be found to have been onely 3 d. 20 min. which is 2 degr. 10 min. leſs then the truth; as in the Plain Chart may be ſeen, where the third Rhumb from the Meridian cuts the Parallel of 55 degr. of Latitude in 3 degr. 20 m. of Longitude at the Point X.

Probl. VIII.

The Difference of Longitude between two Places, the Rhumb leading from one Place to the other, and the Latitude of one of the Places, being given, to find their Diſtance.

The Analogie or Proportion.

As the Sine of the Rhumb from the Meridian is to the Difference of Longitudes;
So is the Radius to the proper Diſtance of the two Places upon the Rhumb.

Thus, if the Latitude of one Place were in 50 degr. the other in a greater Latitude unknown, the Difference of Longitude between the two Places 5½ degr. and the Rhumb N. E. by N. 33 degr. 45 min. from the Meridian; the proper Diſtance upon the Rhumb will be found to be 6 degrees.

Upon the Chart.

LET two Meridians, A B and C D, be drawn through A and C, according to the Difference of Longitude, and a Parallel of Latitude through A, croſſing the Meridian C D in the Point D. Then upon the Point A protract an Angle of 33 degr. 45 min. the quantity of the Rhumb from the Meridian, and draw the Line A C croſſing the Meridian C D in C. So the Diſtance C D, being taken in the Compaſſes, and

measured upon the Meridian-line of the Chart, (respect being had to the Latitude of the Places) that is, so much above the greater Latitude as below the lesser Latitude, you will find it to contain 6 degr.

But if this setting of the Compasses so much above one Latitude as below another seem difficult, it may be thus otherwise done.———For, the Rhumb Line being drawn, it will cut the Meridian C D in C: so a Parallel drawn through C will cut the Meridian A B in B: so is B the Latitude of the second Place, viz. 55 degr. Then divide the Distance between the two Latitudes A and B in two equal parts in the Point M; also divide the Rhumb-Line A C in two equal parts in N: then take the Distance N C or N A, and setting one foot of the Compasses in M, the other will reach to L above the greater Latitude, and from M to K as much below the lesser Latitude, namely, 30 min. or half a Degree on either side; so that between K and L are contained 6 degr. and that is the proper Distance upon the Rhumb.

But if this Distance were to be found by the Plain Chart, it would be almost 10 *degr. or* 197 *Leagues, which is* 77 *Leagues more then in truth it should be. As may appear, if you measure the Line A L in the Plain Chart, upon the Side thereof.*

Probl. IX.

The Difference of Longitude, and Distance of two Places, with the Latitude of one of the Places, being given, to find the Rhumb that leads from one to the other.

The Analogie or Proportion.

As the proper Distance upon the Rhumb is to the Difference of Longitude;
So is the Radius to the Sine of the Rhumb from the Meridian.

Thus

Thus, if one of the Places lay in the Latitude of 50 degr. and the other in a greater Latitude, but unknown; the Difference of Longitude between them 5 ½ degr. and their proper Distance upon the Rhumb 6 degr. the Inclination of the Rhumb to the Meridian which leadeth from one Place to the other will be found to be 33 degr. 45 min. that is the N. E. by N. Point.

Upon the Chart.

LET the Meridians A B and D C be drawn through A and C, and through A a Parallel of Latitude A D. Then open the Compasses (having respect to the Latitudes) from E to L, the quantity of 6 degr. in the Meridian; and setting one foot of that Extent in A, with the other foot cross the Meridian C D in C, and draw the right Line A C for the Rhumb. Lastly, by your Chord or Quadrant find the quantity of the Angle B A C, 33 degr. 45 min, and that is the Rhumb required N. E. by N.

But if you were to find this Rhumb by the Plain Sea-Chart, it would be found almost the E. N. E. Point within a degr. 30 min. differing from truth very near 3 whole Points to the Eastward.

PROBL. X.

The Longitude and Latitude of two Places being given, to find the Distance upon the Rhumb.

The Analogie or Proportion.

As the proper Difference of Latitudes is to the Radius;
So is the Difference of Longitudes to the Tangent of the Rhumb from the Meridian:

And

As the Sine of the Rhumb from the Meridian is to the Difference of Longitude;

So is the Radius to the proper Distance upon the Rhumb.

Thus, the two Places being one in the Latitude of 50 degr. the other in the Latitude of 55 degr. and the Difference of Longitude between them being 5½ degr. the proper Distance upon the Rhumb will be found to be 8 degr.

Upon the Chart.

DRAW the Meridians A B and C D, the Difference of Longitude between them being 5½ degr. and through A and B draw two Parallels B C and A D, and then the Line for the Rhumb leading from the one to the other A C. So A C, being taken in the Compasses, and measured upon the Meridian-line of the Chart, with this Condition, that at the resting of the Compasses upon the Meridian-line, one foot be so many Degrees above the greater Latitude as the other foot is below the lesser Latitude; so will the feet of the Compasses rest in the Points K and L, one being 30 min. below the lesser Latitude, and the other 30 min. above the greater.

And if the Distance upon the Rhumb were to be found by the Plain Chart, it would be found to be about 7 degr. 15 min. or 245 Leagues, which is 25 Leagues more then it should be.

PROB. IX.

The Latitude of two Places and their Distance upon the Rhumb being given, to find their Difference of Longitude.

The Analogie or Proportion.

As the proper Distance upon the Rhumb is to the Radius; So is the proper Difference of Latitudes to the Co-sine of the Rhumb from the Meridian:

And

And

So is the Sine of the Rhumb from the Meridian to the Difference of Longitude.

Thus, if one of the Places be in the Latitude of 50 degr. and the other in 55 degr. and their proper Distance upon the Rhumb 6 degr. or 120 Leagues; their Difference of Longitude will be found to be 5 degr. 30 min.

Upon the Chart.

DRaw A D and B C, two Parallels of Latitude, through 50 degr. and 55 degr. which were the two given Latitudes. Then out of the Meridian Line take the proper Distance upon the Rhumb (having respect to both Latitudes) from K to L: the Compasses being opened to this Distance, one foot being set in A, the lesser Latitude, the other will cross the Parallel of the greater Latitude in C. So the Distance B C, being measured at the bottome of the Chart from E, will reach to 5 degr. 30 min. And such is the Difference of Longitude between the two Places.

But if this Difference of Longitude were to be found by the Plain Chart, it would be but 3 degr. 20 min. which is no less then 2 d. 10 min. less then the truth; as by the Triangle T V. E drawn upon the Plain Chart may appear.

PROBL. XII.

The Difference of Longitude of two Places, their Distance upon the Rhumb, and the Latitude of one of the Places, being given, to find the Difference of Latitudes.

The Analogie or Proportion.

As the proper Distance of the two Places upon the Rhumb is to the Radius;

So is the Difference of Longitudes to the Inclination of the Rhumb to the Meridian:

And

So is the Co-ſine of the Rhumb from the Meridian to the Difference of Latitudes.

Thus, the Difference of Longitudes being 5 ½ degr. their proper Diſtance upon the Rhumb 6 degr. and the Latitude of one of the Places 50 d. the Difference of Latitudes will be found to be 5 d.

Upon the Chart.

THrough the given Latitude A draw a Meridian AB, and a Parallel A D, and upon the Parallel ſet the Difference of Longitude 5 ½ d. taken from the bottom of the Chart, from A to D, and through D draw the Meridian D C. Then out of the Meridian-line take the proper Diſtance upon the Rhumb, 6 d. from K to L, and ſetting one foot of the Compaſſes in A, with the other croſs the Meridian C D in C: ſo a Parallel of Latitude drawn through C will be the Parallel of 55 d. So is 55 d. the Latitude of the other Place, and 50 being taken from 55, leaves 5 d. for the Difference of Latitudes required.

Which Difference, had it been to be found by the Plain Chart, would have been but 2 d. 25 m. that is, 2 d. 35 m. leſs then the truth; as by the Triangle T V E upon the Plain Chart may appear.

PROBL. XIII.

The Latitude of two Places and their Difference of Longitudes being given, to find the Rhumb leading from one to the other, and alſo how many Degrees diſtant they are aſunder.

THIS Propoſition is already performed in the Example of the two Places A and B; but for Variety I will take two other Places, and onely ſhew the manner of working upon the Chart.

Sup-

Suppoſe then two Places, one (as before) in the Latitude of 50 d. the other in the Latitude of 52 degr. 30 min. whoſe Difference of Longitudes is 6 degr.

Upon the Chart.

Through the two given Latitudes 50 d. and 52 ½, at A and O draw two Parallels, O P and A D, upon which ſet the Difference of Longitudes from O to P, and from A to Q, 6 degr. Then draw the Line A P, which ſhall be the Line of the Rhumb leading from one Place to the other: wherefore, by your Chord or Protracting Quadrant find the quantity of the Angle O A P, which ſhall be the Inclination of the Rhumb to the Meridian, and will be found to be 56 d. 15 m. that is the N. E. by E. Point; which was the Firſt thing that was required.

Then to find the proper Diſtance upon the Rhumb; Take the Line A P in your Compaſſes, and meaſure it upon the Meridian-line, ſo that one foot may be above the greater Latitude ſo much as the other is below the leſſer; and you will find the Compaſs-points to reſt in E and S, E being one whole Degree below the leſſer Latitude, and S one Degree above the greater. So that there is intercepted between E and S 4 ½ degr. And that is the proper Diſtance upon the Rhumb; which was the Second thing required.

But if this Problem had been wrought upon the Plain Chart, the Rhumb from the Meridian would be found to be 67 d. 23 m. that is, within 7 m. of the 6th Rhumb; which is more then the truth by 11 d. 8 m.

Probl. XIV.

A Ship ſet ſail from the Latitude of 50 degr. upon the fifth Rhumb N. E. by E. after that ſhe had made 36 Leagues of way upon that Rhumb, the wind changing, ſhe was conſtrained to ſail 50 Leagues upon the 7th Rhumb E. by N. I would know in what Longitude and Latitude the Ship is.

Upon

Upon the Chart.

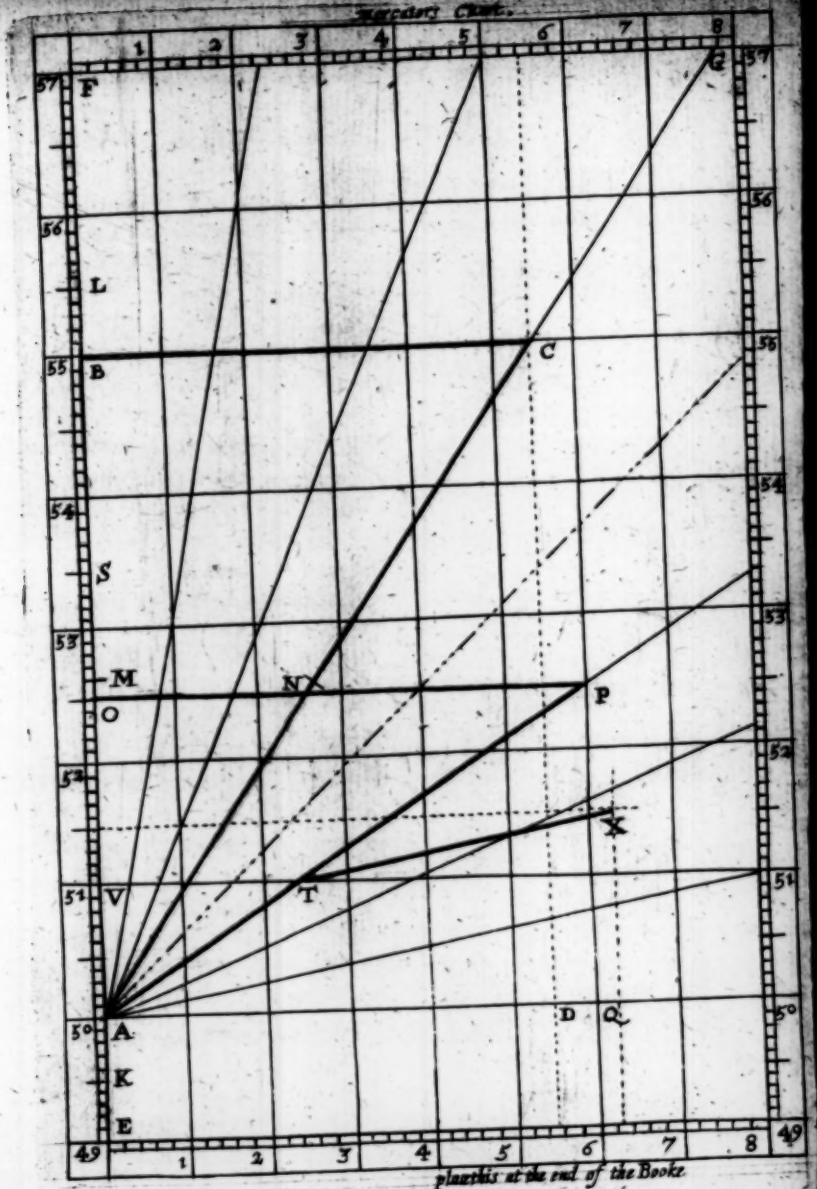
THE Rhumb-Line A P being drawn, ſet off thereupon 36 Leagues (which was the way that the Ship made upon the fifth Rhumb before the Wind changed) from A to T, (which Diſtance muſt be taken out of the Meridian-line by opening the Compaſſes from 50 d. to 51, 48. or better, to as much below 50 d. as above 51 d.) So ſhall the Point T be the Place that the Ship was in when the Wind altered. So a Parallel drawn through T upon the Chart will cut the Meridian at V in 51 d. and in that Latitude the Ship was. Now to find in what Longitude ſhe was; Take in your Compaſſes the Line T V, and meaſure it at the bottom of the Chart, you ſhall find it will reach from E to 2 d. 21 m. And in that Longitude the Ship then was.

This done, upon the Point T (where the Wind changed, and drove the Ship 2 Points more Eaſtwardly, namely, upon the E. by N. Point) protract an Angle of 22 d. 30 m. namely, the Angle P T X, which is the Rhumb upon which the Ship ſailed 50 Leagues after the Wind changed. Therefore take 50 Leagues out of the Meridian-line, and ſet them from T to X. So ſhall X be the Place that the Ship was in after ſhe had ſailed 50 Leagues upon the E. by N. Point; which, by drawing a Parallel through K, will be found in the Latitude of 51 d. 30 m. and by drawing of a Meridian through K alſo, it will be found to be in the Longitude of 6 degr. 16 min.

But if theſe Courſes had been protracted according to the Plain Sea-Chart, the Point T would fall in the Latitude of 51 *degr. and the Point X in the Latitude of* 51 *degr.* 30 *m. But the Longitude of T would be onely* 1 *d.* 30 *m. and the Longitude of X in* 3 *d.* 57 *min. Both theſe Longitudes being added, make but* 5 *d.* 27 *m. for the Difference of Longitude between X and the firſt Meridian; whereas by the other Chart it is* 6 *d.* 16 *m. So that the Ship at X is* 33 *m. Weſtward of the Place to which ſhe was bound.*

Theſe Differences, which I have obſerved to be between the Plain and Mercator's *Chart, may be ſeen by comparing the Scheme of the two Charts together.*

FINIS.


Mercators Chart.
1
2
3
4
5
6
7
8
57
56
55
54
53
52
51
50
49
F
L
B
S
M
O
V
A
K
E
G
C
N
P
X
T
D
Q
plaethis at the end of the Booke